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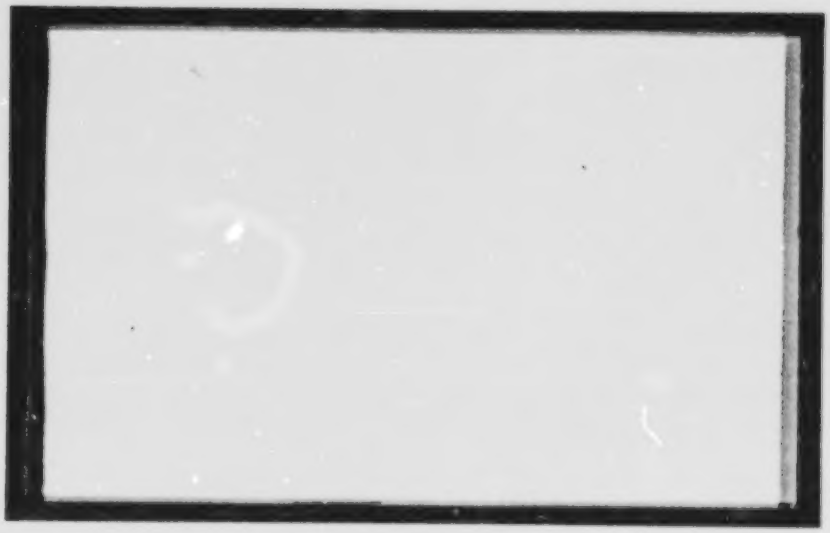

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A METHOD FOR SCHEDULING URBAN TRANSIT VEHICLES WHICH TAKES ACCOUNT OF OPERATING LABOR COST

by

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Abstract

A method for scheduling urban transit vehicles which takes account of operating labor cost

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Carnegie-Mellon University, 1984

Abstract

As an exact formulation of the transit vehicle scheduling problem cannot be solved using existing technology, the problem is always decomposed in practice into more manageable components. The traditional decomposition performs vehicle-scheduling prior to manpower scheduling and takes only scant account of the cost of manpower at the time vehicle schedules are made. This research develops a vehicle scheduling technique which uses a linear manpower cost estimation model to generate vehicle schedules which take into account the cost of supplying manpower. The linear program is an extension of that used in the HASTUS system (Rousseau, Lessard, and Blais, 1983) for manpower scheduling in public transit, and serves as a mathematical model for estimating the efficiency which which alternate vehicle solutions can be served with manpower. The method starts with an initial vehicle solution as a point of departure and modifies this initial solution, using a heuristic procedure, according to the indications of potential manpower cost savings which arise from the linear model.

We apply the new method to trip timetable data available from the public transit operating agency in Montréal, Québec and estimate that the potential

transit operating agency in Montréal, Québec and estimate that the potential percentage savings in manpower cost from making any changes to the vehicle solution are slight. The vehicle schedule improvement heuristic, however, produces a feasible vehicle solution which involves relatively few changes to the initial solution and yet which implies a decrease in estimated manpower cost.

The solution to the dual of the linear estimation model has a natural interpretation as a set of marginal costs associated with of small additions to or reductions in service. The fact that the linear program is often primal degenerate means that many different optimal dual solutions can exist. We minimize a new quadratic objective function in the dual variables to arrive at a unique dual solution which has several desirable properties. In applying this method to the Montréal data, we find that the estimated marginal cost of small service changes in the periods before the morning peak and after the evening peak are quite high, even though these periods are normally thought of as "off-peak".

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Chapter 1

Introduction and overview

1.1 General introduction

As is the case with many complex operational problems, public transit vehicle and manpower scheduling must be broken down into discrete stages to be manageable. The usual procedure in transit, once transit trips have been established in a timetable, has been to create vehicle schedules first and manpower schedules only when vehicle schedules are known (Keaveny and Burbeck, 1981). This standard procedure has the advantage that the two scheduling functions are much simpler to carry out individually than together. In fact, an exact optimal solution to the entire transit scheduling problem is beyond the capabilities of modern technology (Bodin, Golden, Assad, and Ball, 1983). But the ultimate cost of providing transit service will depend on the way manpower is assigned to cover a set of vehicle schedules, because operating labor must almost always be assigned duties in accordance with a complex system of limits and premium payments (Sharp, 1975). Since some of these limits and premiums may depend on aspects of a vehicle schedule solution, the traditional decomposition of transit scheduling may introduce a certain inefficiency into the scheduling process.

This research introduces manpower cost information into the scheduling of transit vehicles. The approach consists in two major steps. We first find a feasible vehicle schedule solution by solving a transhipment model which minimizes the total time in that solution. We then attempt to improve the initial solution from the point of view of manpower cost by applying a linear cost model derived from that used in the HASTUS transit manpower scheduling

system (Rousseau, Lessard, and Blais, 1983). The linear manpower cost model is adapted to the vehicle scheduling problem by the addition of rows and columns which do not appear in the original formulation of HASTUS.

The fact that this research uses a linear programming relaxation of the transit vehicle scheduling problem gives rise to an opportunity to interpret the solution to the dual linear program as a set of marginal costs of changes to the level of transit service at various times of the day. Because of degeneracy in the primal linear program a meaningful direct interpretation of the dual variables is difficult. This research develops a method for applying additional criteria to the dual problem in order to yield a unique, meaningful dual solution.

Finally, we apply the methods developed in this research to trip time-table data available from the public transport agency in Montréal, Québec (la Commission de transports de la Communauté Urbaine de Montréal).

This chapter introduces the subject matter of this dissertation. Section 1.2 is a general introduction to transit scheduling. Section 1.3 discusses the ways in which the scheduling of manpower introduces nonlinearity into the relationship between vehicle time and manpower cost, and hence why the information gap between vehicle and manpower scheduling has any importance at all. Finally, Section 1.4 gives an overview of the organization of the dissertation.

From time to time certain technical transit terms will be introduced. These terms are underlined at the time of their introduction and are defined in the text. A glossary of all such terms is included in this dissertation as Appendix A.

1.2 An introduction to transit scheduling

Keaveny and Burbeck (1981) give an overview of the transit scheduling process as it exists at almost all North American transit agencies, or properties. The process begins when service requirements are determined from ridership counts or other demand information. These service requirements are made concrete in a trip schedule, which is the same as a public timetable. A trip schedule lists a set of trips, or vehicle movements between termini in a single direction, which operate on a single line.

The trips are assigned to transit vehicles in a vehicle scheduling step which generates a set of vehicle schedules, or blocks. Each vehicle schedule is a sequence of trips which defines the task of a single vehicle between the time it leaves the garage and the time it returns. Thus the vehicle schedules exhaustively partition the set of trips in the trip schedule.

According to Keaveny and Burbeck (1981), schedulers using traditional manual techniques often found that the most convenient method of creating vehicle schedules involved writing trip and vehicle schedules simultaneously because the scheduler was then sure that trips could be hooked, or put into sequence in a vehicle schedule, without unduly long layovers, or waiting time at terminals. Recently, however, trip and vehicle schedule writing are becoming more distinguishable as some properties are finding that the practice of interlining can reduce the number of vehicles required to serve a set of revenue trips. A property which performs interlining combines revenue trips from two or more trip schedules into a single pool of trips. Trips from this pool are considered together in forming vehicle schedules at any one time, and the vehicle scheduling process covers all such trips in the pool with vehicles.

The practice of interlining is made widely possible by computerized

scheduling techniques. Since empty running or deadheading times between time points on different lines are an important component of the process, data management can become cumbersome without a computerized transit scheduling management information system. In addition, the least-cost hooking scheme is usually not apparent from visual inspection of a large pool of trips on several lines. Program packages such as the SAGE MiniScheduler (SAGE Management Consultants, 1982) and RUCUS-II (Luedtke, 1983) maintain files of deadheading times and are designed to schedule vehicles automatically using a minimum cost flow model. The model used by these particular packages to solve this problem is described in Chapter 2 of this dissertation.

The traditional scheduling process continues with the task of manpower scheduling once vehicle schedules have been formed. This task is also called runcutting since vehicle blocks are "cut" into pieces of work which are then assigned to operators. Despite the currency of the term "runcutting" in the transit industry, there is more to manpower scheduling in transit than just cutting, since daily operator assignments are often combinations of two or more pieces of work and the assignment of pieces of work to operator duties is a nontrivial combinatorial problem in itself (Rousseau, Lessard, and Blais, 1983; Ball, Bodin, and Greenberg, 1983). For this reason and because certain authors (for example, Bodin, Rosenfield, and Kydes, 1981) use the term "run" to mean a transit trip instead of the daily work of an operator, we avoid its use in favor of "assignment" or "duty".

Vehicle schedules can only be cut and operators relieved at time points representing a vehicle's passage of a relief point. Relief points may be in areas where, for example, meal facilities are provided or where there is connecting transit service back to the garage.

Figure 1.1 illustrates the relationship between trips, blocks, and relief

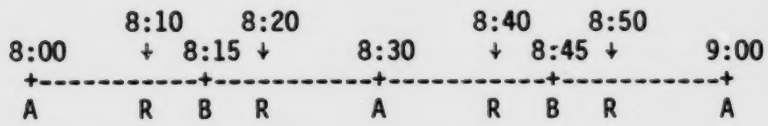
Figure 1.1

The positioning of relief points
in a vehicle schedule

Consider 4 trips:

1. Leave A at 8:00, arrive B at 8:15;
2. Leave B at 8:15, arrive A at 8:30;
3. Leave A at 8:30, arrive B at 8:45;
4. Leave B at 8:45, arrive A at 9:00.

These go together into a single vehicle schedule:



The relief point R is 10 minutes from A , 5 minutes from B . Thus the operator can be relieved at 8:00 (start of block), 8:10, 8:20, 8:40, 8:50, and 9:00 (end of block).

points. On a fictitious transit line between two points A and B , the time to go either from A to B or from B to A is 15 minutes. The relief point R is 10 minutes' running time from A and 5 minutes' running time from B . We can consider several trips on this fictitious line which have been strung together into a vehicle schedule. The relief times, or times at which an operator can be relieved, are determined by the relative position of the relief point, as is illustrated in Figure 1.1.

Sharp (1975) has reviewed the major types of manpower scheduling constraints and premiums in transit. Table 1.1 lists these contraints and premiums by category. A major distinction exists between those which apply to individual duties and those which apply property-wide. Conditions applying to individual duties can be further classified into limits (absolute maxima or minima which cannot be violated in any duty), premiums (extra pay to compensate for certain working conditions), guarantees (minimum amounts considered worked, regardless of the amount of time actually worked), and allowances (standard pay, usually small, to compensate for certain non-driving duties of operators). Conditions applying property-wide generally take the form of maximum or minimum percentages of total operator duties.

The complexity of transit manpower scheduling provided early incentive for the development of computerized manpower scheduling tools, which apparently began with Elias (1960). Researchers have searched for a way to apply known techniques to transit manpower scheduling formulated as an exact optimization problem, but have found that the needs of even moderately-sized transit properties generate problems the solution of which is far beyond the capabilities of existing computers (Bodin, Golden, Assad, and Ball, 1983).

Wren (1981) and Rousseau (1983) review available transit manpower scheduling solution methods. These approaches generally accept vehicle

Table 1.1

Representative scheduling conditions found
at North American transit properties

INDIVIDUAL DUTIES

Limits

Straight time:	Number of hours after which overtime must be paid.
Spread:	Maximum time between initial sign-on and final sign-off.
Spread premium limit:	Time limit after which premium pay for long spread applies.
Piece size (maximum):	The longest time an operator can be required to be on duty without a break (exclusive of layovers).
Piece size (minimum):	The shortest such time.
Unpaid break (maximum):	The outer limit of a break in the middle of the day for which the operator is not paid -- any break longer than this must include some paid time.
Unpaid break (minimum):	The shortest break which can be unpaid (to avoid very short intervals which are not paid).
Maximum number of unpaid breaks per day.	
Time of day restrictions:	Restrictions on sign-on times, sign-off times for duties of various sorts.

Premiums

Overtime:	Premium paid for time worked after straight time limit.
Spread:	Premium paid for time worked after spread limit.

Table 1.1 (continued)

Guarantees	
Piece:	Minimum amount paid for any piece of an operator duty.
Daily total pay:	Minimum amount paid for any day's work.
Daily platform time:	Minimum platform time for any daily assignment.
Allowances	
Travel:	Amount allowed for travel between relief points or to/from the garage.
Turn-in and report:	Vehicle preparation, check-out, and check-in at the beginning and end of a working day.
AGGREGATE DUTIES	
Percentages	of duties with or without an unpaid break.
	of part-time operators.

Note: Adapted from material in Sharp (1975).

schedules as given, and operate in one of three ways. Some generate feasible duties according to simple heuristics, without benefit of any global optimization algorithm. This is the approach of RUCUS (Roberts, 1974) and its successors, notably RUCUS-II (Hildyard and Wallis, 1981; Luedtke, 1983). Others solve an optimization problem abstracted from the original scheduling problem to find a region in which an eventual "good" solution will be found and then solve additional subproblems to generate an actual feasible schedule in this region. An example of this approach is the HASTUS system (Rousseau, Lessard, and Blais, 1983). A third approach sets up and solves the manpower scheduling problem as the set-covering problem (Wren, Smith, and Miller, 1981; Marsten and Shepherdson, 1981). This approach has proven to be effective in situations involving relatively few operators, as problem size depends on the number of operators to be scheduled and size limitations can arise.

A fourth approach, which is still in experimentation, is that of Ball, Bodin, and Dial (1983). These authors radically redefine the transit scheduling process to carry out vehicle and manpower scheduling as a unified process. This procedure is discussed in more detail in Chapter 2.

The complexity of transit manpower scheduling is such that a fully satisfactory assignment of vehicle time to operators cannot always be achieved and some small leftover pieces of work often remain at the end of scheduling. These leftover pieces, or <u>trippers</u>, must either be worked in overtime by regular operators or by <u>extraboard</u> operators who are normally paid to stand by in case a regular operator is not available for his assignment. The use of such personnel to handle trippers is generally regarded within the industry as inefficient because an extraboard operator is usually paid a daily guarantee regardless of whether he works or not and it is often not possible to find enough work to occupy such an operator for more than a small portion of his

paid time.

Finally, weekly operator assignments must be created in such a way that operators are allowed an appropriate number of days off. The scheduling rules which apply to this weekly assignment process, or rostering, can make it an interesting optimization problem in itself. See Bennett and Potts (1968) and Rousseau (1983) for some approaches to the rostering problem.

1.3 The nonlinearity of labor cost with respect to vehicle time

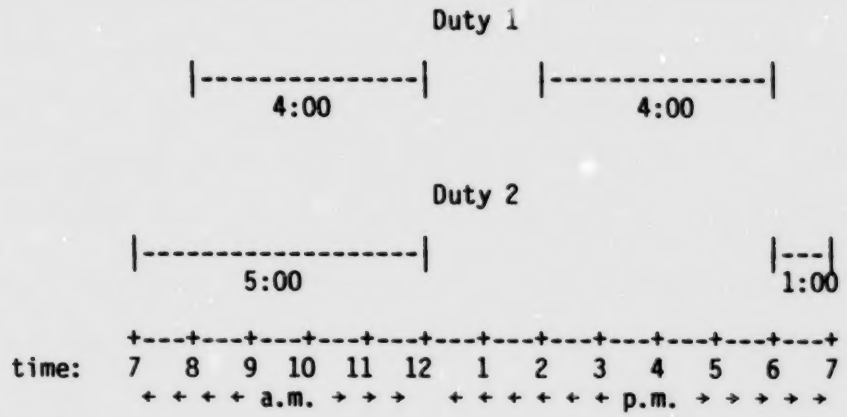
The actual cost of serving transit vehicle schedules is not known until a set of operator assignments has been created to cover these vehicle schedules. Yet as this section describes, the rigors of manpower scheduling can mean that the length and arrangement in time of vehicle schedulies can affect the amount paid in manpower per unit of vehicle time. The implications of this phenomenon for transit is that vehicle scheduling is carried out with incomplete knowledge of the cost implications of any particular solution.

The presence of an overtime premium is sufficient to introduce nonlinearity into the relationship between vehicle time and manpower cost, as then some vehicle hours are actually covered with manpower at a higher cost than others. But the fact that many properties have to pay a piece guarantee, or minimum pay for any piece of an operator assignment no matter how long, as well as a spread premium, or extra pay for workdays which span long periods of time, can actually make this relationship nonuniform. To see this, consider the hypothetical manpower scheduling situation of Figure 1.2. A transit property pays each operator a minimum of eight hours per day and three hours per piece, and pays a spread premium of time and one-half after ten hours of spread, or the time from the very beginning to the very end of the workday. The property has created four vehicle schedules and wishes to create two

Figure 1.2

Example of the dependence of manpower cost
on piece guarantee and spread

Work rules: Eight-hour day guarantee
Three-hour piece guarantee
Spread premium of one-half time after ten hours



Duty 1 is paid 8:00, as no guarantees or premiums apply

Duty 2 is paid 9:00, as follows:

6:00 time worked
2:00 guarantee time
1:00 spread premium for two hours after spread limit of 10:00

duties, each comprising two pieces of work. Duty 1 involves eight hours' work spread over ten hours. Thus the total pay for this duty is exactly eight hours: no guarantees or premiums apply. Duty 2, however, involves six hours' work spread over twelve hours. The total pay for this duty is nine hours: two hours of piece guarantee for the second piece (or, equivalently, two hours of daily guarantee for the entire workday) and one-half hour of spread premium for each of the two hours beginning at 5:00 p.m., which is exactly ten hours after the beginning of the assignment. Note that a spread premium would even apply to an hour (between 5:00 p.m. and 6:00 p.m.) during which no work is performed at all. Thus as a result of these premiums and guarantees the first duty pays eight hours for eight hours of vehicle time, while the second duty pays nine hours for only six hours of vehicle time. Similarly, the average cost per vehicle hour covered by duty 1 is one hour of pay, while the average cost per vehicle hour covered by duty 2 is one and one-half hours of pay.

Despite the divergence in average labor cost to serve vehicle time in this example, the traditional transit scheduling process discussed in Section 1.2 contains no way to take such a phenomenon into account in constructing vehicle schedules. As we shall see in the course of this dissertation, however, the way in which vehicle schedules are made can in fact influence the amount of permiums paid in scheduling manpower.

1.4 Dissertation overview

This dissertation is organized in the following manner. Chapter 2 is an overview of currently available approaches to transit vehicle scheduling and service costing. Special attention is given to computerized scheduling methods which are available, either commercially or from the United States Department of Transportation, to transit agencies at the present time.

Chapter 3 presents the linear program now used by the HASTUS manpower scheduling system and the extended linear program developed by this research. Chapter 4 constructs an example of the problem matrix for the linear programming model of Chapter 3. Chapter 5 is concerned with a heuristic method for using the linear model to create feasible vehicle schedules which imply a more efficient use of manpower. Chapter 6 presents numerical results obtained from applying the new method to trip timetables in actual operation in Montréal, Québec. Chapter 7 describes a technique for obtaining a unique solution to the dual of the linear program of Chapter 3 and gives some numerical results. Chapter 8 gives a summary and some conclusions and proposes some directions for further research.

Chapter 2

Currently available vehicle scheduling and service costing methods for transit

2.1 Introduction

This research develops a method of transit vehicle scheduling using a linear program and also investigates the dual of this linear program as a means to estimate the marginal cost of transit service. Accordingly, this review of currently available methods is divided into two parts. The first summarizes current approaches to transit vehicle scheduling, and the second gives an overview of available transit service costing methods.

2.2 Transit vehicle scheduling methods

This research presents a solution to what Bodin, Golden, Assad, and Ball (1983) call the simple transit vehicle scheduling problem (VSP). The VSP seeks to optimally assign vehicles to a fixed set of trips which make up a public transit timetable. Each trip is to be served by a vehicle once and only once. The problem assumes, in addition, that there are no restrictions on the length of time during which a vehicle can be away from the garage, that all trips can be served by the same kind of vehicle, and that all trips are served by vehicles from the same garage.

One of the most widely-known solution methods for the VSP uses a model of minimum cost flow on an acyclic graph. There is a fairly standard formulation of this problem which appears frequently in the literature and in commercial scheduling software packages. This section takes a very close look at this method, which is here given the name VSP1, for two reasons. First, an

improvement upon VSP1, known as VSP2, will serve in this research as a first approximation to a vehicle schedule solution. Second, VSP1 is still widely regarded as the classic solution to the VSP (Bodin, Golden, Assad, and Ball, 1983). It will therefore be of interest to closely examine some of its attributes and shortcomings.

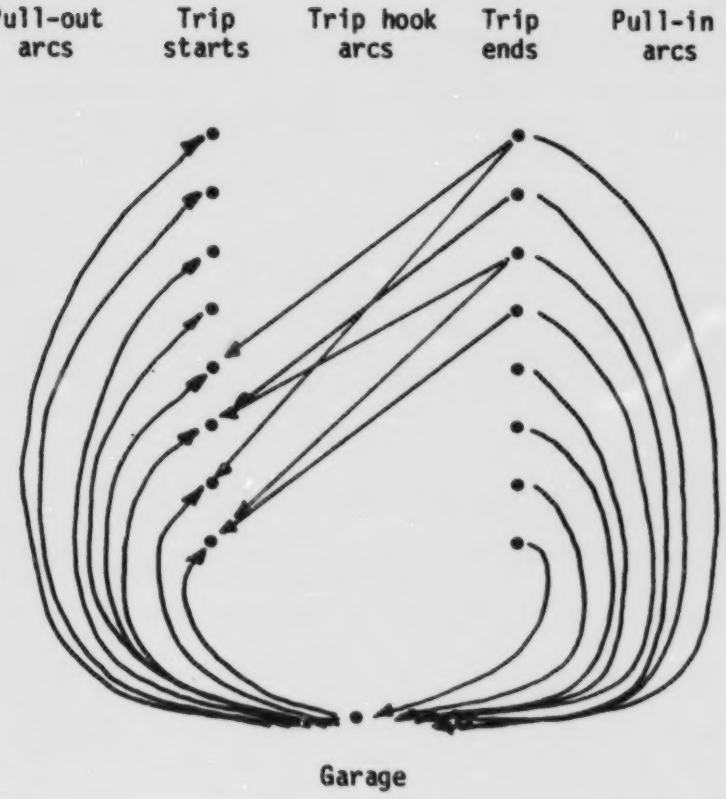
2.2.1 VSP1: a common method. The simple VSP can be formulated as a transhipment problem, as follows. Assume a timetable of N trips, each characterized by a 4-tuple $\{ t_i, u_i, l_i, m_i \}$, where t_i is the start time, u_i is the end time, l_i is the start location, and m_i is the end location for trip i. Associate with each trip i two nodes: a source node U_i with a supply of one representing the end of trip i and a sink node T_i with a demand of one representing the beginning of trip i. In addition, define a transhipment node G which represents the bus garage at which all vehicle schedules must begin and end.

Define a set H of ordered pairs (i,j) where $i,j \in N$. A pair $(i,j) \in H$ if trip j can feasibly follow trip i in some vehicle schedule, or that trip j can feasibly be hooked to trip i. The existence of a pair (i,j) in H means that there is an arc from U_i to T_j in the graph. There are also arcs in the graph from G to each T_j and from each U_i to G. Figure 2.1 is a schematic diagram of a vehicle scheduling problem set up using this transhipment formulation.

The objective function to be minimized in the simple VSP is very often a composite of a number of variables, of which three arise frequently: the size of the vehicle fleet required to service the timetable, the requirement of

Figure 2.1

Schematic diagram of a VSP1
solution graph



Note: Trip end nodes each have a supply of one, trip start nodes each have a demand of one.

platform time, or total time spent by vehicles away from the garage, and the total distance travelled by vehicles. This kind of objective function has been used, with variations, in several approaches to automated transit scheduling such as the RUCUS BLOCKS package (Bennington and Rebibo, 1975), RUCUS-II (Luedtke, 1983), and the solution of Gavish, Schweitzer, and Shlifer (1978). Orloff (1976) and Bodin et al (1983) describe models which use a similar cost structure. RUCUS and RUCUS-II are software packages available to public transit agencies from the United States Department of Transportation.

Note that there is no explicit representation of manpower cost in such an objective function. In a sense, the platform time component serves as a surrogate for manpower cost under an assumption that the relationship between platform time and manpower cost is more or less linear.

The use of a composite objective function permits the vehicle scheduling program to make implicit trade-offs between various aspects of a vehicle schedule, as minimizing the fleet size is not always the sole objective in vehicle scheduling and some users may prefer a slightly larger fleet if this reduces the occurrance of some other factor. For example, the practice of interlining, or switching vehicles between trips on different lines, is often discouraged for a variety of institutional reasons even though occasional interlining may in fact reduce the size of the bus fleet or save some platform time. A composite objective function might include a penalty on the inclusion of trips from different lines in the same vehicle schedule.

A composite objective function including components for platform time, vehicle miles, and bus fleet size could be established as follows. For each $(i,j) \in H$, define an arc cost c_{ij} as

$$c_{ij} = T(t_j - u_i) + Kd(i,j)$$

where $d(i,j)$ is the distance from m_i to l_j and T and K are appropriate constants. Under this definition, $c_{ij} > 0$ for all feasible $(i,j) \in H$. If $m_i = l_j$, so that $d(i,j) = 0$, $c_{ij} > 0$ is implied simply by $t_j < u_i$. If $d(i,j) > 0$, let $h(i,j)$ be the time to move the vehicle empty, or <u>deadhead</u>, from m_i to l_j. In this case, $(i,j) \in H$ only if $t_j > u_i + h(i,j)$. The quantity

$$t_j - u_i - h(i,j)$$

is the time the vehicle waits at l_j before starting trip j at time t_j and is called a <u>layover</u>.

Now define c_{Gj} as the cost of beginning a vehicle schedule, or of performing a <u>pull-out</u> from the garage, at trip j:

$$c_{Gj} = Th(G,j) + Kd(G,j) + L$$

where $h(G,i)$ is the deadhead time from the garage to l_j, $d(G,j)$ is the deadhead distance from the garage to l_j, and L is a constant representing the incremental cost of adding an additional vehicle to cover service in the timetable. Similarly, define c_{iG}, the cost of performing a <u>pull-in</u> or ending a vehicle schedule, at trip i as

$$c_{iG} = Th(i,G) + Kd(i,G)$$

where $h(i,G)$ and $d(i,G)$ are defined analogously to $h(G,j)$ and $d(G,j)$.

Define a series of binary zero-one variables x_{ij} for $(i,j) \in H$, x_{iG} for $j \in N$, and x_{Gj} for $j \in N$ as follows:

$x_{ij} = \begin{cases} 1 & \text{if trip } j \text{ follows trip } i \text{ in a vehicle schedule} \\ 0 & \text{otherwise} \end{cases}$

$x_{iG} = \begin{cases} 1 & \text{if trip } i \text{ is the last trip in a vehicle schedule} \\ 0 & \text{otherwise} \end{cases}$

$x_{Gj} = \begin{cases} 1 & \text{if trip } j \text{ is the first trip is a vehicle schedule} \\ 0 & \text{otherwise} \end{cases}$

We then pose the problem

$$\text{VSP1: min} \quad \sum_{(i,j)\in H} C_{ij}x_{ij} + \sum_{i\in N} C_{iG}x_{iG} + \sum_{j\in N} C_{Gj}x_{Gj} \qquad (2.1)$$

$$\text{subject to} \quad \sum_{i\in N} x_{iG} - \sum_{j\in N} x_{Gj} = 0 \qquad (2.2)$$

$$\sum_{i:(i,j)\in H} x_{ij} + X_{Gj} = 1 , \quad j \in N \qquad (2.3)$$

$$\sum_{j:(i,j)\in H} X_{ij} + X_{iG} = 1 , \quad i \in N \qquad (2.4)$$

$$0 \leq X_{ij} \leq 1 \quad \text{and integer for all} \quad (i,j) \in H$$

$$0 \leq X_{iG} \leq 1 \quad \text{and integer for all} \quad i \in N \qquad (2.5)$$

$$0 \leq X_{Gj} \leq 1 \quad \text{and integer for all} \quad j \in N$$

Constraint (2.2) is a flow conservation constraint on the transhipment node G , which assures that the total number of pull-outs will be equal to the total number of pull-ins. Constraint (2.3) says that total flow into node

T_i , either from any of the U_j or from G , must be unity for all $i \in N$. This constraint is interpreted to mean that in a feasible solution each trip is either hooked to another trip or immediately precedes a pull-in. Similarly, constraint (2.4) says that total flow out of node U_j , either to any of the T_i or to G , must be unity for all $j \in N$. Thus each trip must be either hooked from another trip or must immediately follow a pull-out.

The use of VSP1 presents certain pitfalls, however. The peaked nature of public transit demand almost ensures that certain transit vehicles will be out of service during the middle of the day, to avoid unproductive operator time during this period. Unfortunately, VSP1 has a built-in tendancy to avoid any solution involving out-of-service time for vehicles. The penalty L on the entry of vehicles into service, which is intended to represent the cost of adding expensive equipment to the system, would be set much larger than the costs c_{ij} on trip hooks, which are a function only of minutes of vehicle time and deadhead distances. Thus VSP1 will naturally favor even very long layovers and deadheads over pull-outs and pull-ins, even though the same vehicle pulled out a second time in the same day does not represent additional capital investment.

Bodin et al (1983) recognize this problem with VSP1, but do not propose a definitive solution to it. They do suggest, however, that the penalty L only be applied to arcs representing a pull-out before a certain hour of the day, such as 12:00 noon. The success of such a solution, however, would depend on the trips being scheduled. For example, if more vehicles were needed to serve a schedule in the afternoon than in the morning then this solution would still not guarantee that VSP1 would function properly.

The approach of Gavish et al (1978) and commercial applications of VSP1

such as RUCUS-II (Luedtke, 1983) allow the analyst to specify a maximum feasible layover after any trip. These exogenous limits on layover have the effect of cutting down on excessive layovers by simply excluding any solution which would involve any. Note, however, that in principle artificially restricting the number of solution possibilities adds additional problem constraints and one should never be able to improve a solution by constraining the solution space.

Despite the popularity of the VSP1 model, there is another way to formulate the VSP which takes more adequate account of the fact that the same vehicle may be used more than once in the same day. We will develop this improved model, which we will call VSP2, in the next section and we will make use of it subsequently in obtaining a preliminary vehicle schedule solution during the course of this research.

2.2.2 VSP2: an improved method. A new model for solving the simple VSP with a composite objective function which seems to correct the major difficulty with VSP1 has been developed by Desrosiers, Soumis, and Desrochers (1982) using a modification of a network formulated by Bokinge and Hasselstrom (1980). As in VSP1, let U_i , $i \in N$ be a set of nodes representing trip ends, and let T_j , $j \in N$ be a set of nodes representing trip starts. We place a supply of unity on each of these U_i and a demand of unity on each T_j . Now, however, we add $2N$ garage nodes as follows. For each $i \in N$, let

$$g_i = t_i - h(G,i) \tag{2.13}$$

$$h_i = u_i + h(i,G) \tag{2.14}$$

That is, g_i is the time at which a vehicle leaving the garage to serve trip i would have to do so, and h_i is the time at which a vehicle entering the garage after serving trip i would have to do so. Now combine all the g_i and h_i into the same set E. We give each element of E an alias e_k, where the $2N$ indices k are such that $k < k'$ if and only if $e_k < e_{k'}$. (In the case of equality of two elements of E the choice of k and k' is arbitrary.) We then associate with each e_k a garage node G_k on which we place neither supply nor demand.

The use of these $2N$ garage nodes distinguishes this approach from that of Bokinge and Hasselstrom (1980), who use garage nodes defined from a discretization of the service day. We will see, in addition, that a node aggregation procedure can reduce the number of required garage nodes substantially without reducing the amount of information in the model.

Using these $2N$ garage nodes, we place a pull-out arc from G_k to T_j if $e_k = g_j$ for $j \in N$ (e_k is an alias for g_j) and a pull-in arc from U_i to G_k if $e_k = h_i$ for some $i \in N$ (e_k is an alias for h_i). In other words, each node T_j is at the head of a pull-out arc from a unique garage node and each node U_i is at the tail of a pull-in arc to another unique garage node.

We define a cost

$$c_{Gj} = Th(G,j) + Kd(G,j) \quad , \quad \text{for} \quad j \in N$$

applicable to the pull-out arc to T_j and a cost

$$c_{iG} = Th(i,G) + Kd(i,G) \quad , \quad \text{for} \quad i \in N$$

applicable to the pull-in arc from U_i. The pull-in arc costs used here are

identical to those used in VSP1 but the pull-out arc costs involve no fleet size penalty L. We place a capacity of one on all garage arcs.

We also place an arc from G_k to G_{k+1} for $k = 1,\ldots,2N-1$, each with a nominal capacity of N, the number of trips, and a cost of zero. We place an arc from G_{2N} to G_1, also with a nominal capacity equal to N and a cost equal to the fleet size penalty L. Note from this that there is a path from every garage node to every other garage node, and a path from G_k to $G_{k'}$ where $k' > k$ which involves no cost. Also note that the nominal capacity N will never be exceeded even if all these arcs were uncapacitated because in an extreme case each trip would be served by its own vehicle and only one vehicle is required to serve any trip.

We define the same inter-trip hook cost function

$$c_{ij} = (t_j - u_i) + Kd(i,j)$$

as was used in VSP1 but we redefine the set H of (i,j) pairs. Now we restrict membership in H to only those pairs (i,j) where

$$c_{ij} < c_{iG} + c_{Gj} .$$

In other words, only those trip hooks which cost less than the sum of the costs on the corresponding pull-in and pull-out arcs are considered in this problem. The reason for this restriction is simple. Any hook (i,j) such that

$$c_{ij} > c_{iG} + c_{Gj} \qquad (2.15)$$

will never be found in an optimal solution, because it will always be less expensive to satisfy the demand at T_j and the supply at U_i with a pull-in to G_k where $h_i = e_k$ and a pull-out from $G_{k'}$ where $g_j = e_{k'}$. We know

that such a situation is feasible because from the definitions of c_{ij}, c_{iG}, and c_{Gj} we have

$$\begin{aligned} T(t_j - u_i) + Kd(i,j) &> Th(i,G) + Kd(i,G) + Th(G,j) + Kd(G,j) \\ &= T[h(i,G) + h(G,j)] + K[d(i,G) + d(G,j)] \end{aligned} \qquad (2.16)$$

By the triangle inequality,

$$d(i,G) + d(G,j) > d(i,j)$$

so that the distance terms can be dropped from (2.16) without disturbing the inequality. Dividing both sides of (2.16) by T, we have

$$t_j - u_i > h(i,G) + h(j,G)$$

or

$$t_j - h(G,j) > u_i + h(i,G)$$

But we can apply the definitions (2.13) and (2.14) to yield that

$$g_j > h_i$$

Thus we have that $h_i = e_k$ and $g_j = e_{k'}$ where $k' > k$, so that there is a costless path from node G_k to node $G_{k'}$, and hence a path from U_i to T_j which passes through G_k and $G_{k'}$. Moreover, the total cost of this path is $c_{iG} + c_{Gj}$.

Note that the reason for limiting membership in H in this problem is not the same as the reason for doing so in commercial applications of VSP1 such as RUCUS-II. In this problem we eliminate long layover arcs which will never carry flow in an optimal solution. In VSP1 long layover arcs are eliminated <u>because</u> in fact they might carry flow in an optimal solution and the analyst wants to discourage this.

We present, in Figure 2.2, a schematic diagram of a scheduling problem set up using the present graph. The horizontal axis in Figure 2.2 represents time during the service day, but the vertical axis is without significance. Note that flow is from left to right except on the single return arc at the bottom.

There is a way to reduce the number of garage nodes G_k, however, without loss of any information in the mode. The method is the same as that applied by Soumis (1978) in the context of aircraft scheduling. The first two garage nodes at the left of Figure 2.2 have in common the fact that each is at the tail of a pull-out arc. We could easily replace these two nodes by another, called G_1^*, which is at the tail of both pull-out arcs. We could not, however, include the third garage node in this aggregation of nodes because this node is at the head of a pull-in arc. If we did include it, there would be nothing to prevent flow from a trip to itself, which would not make any sense.

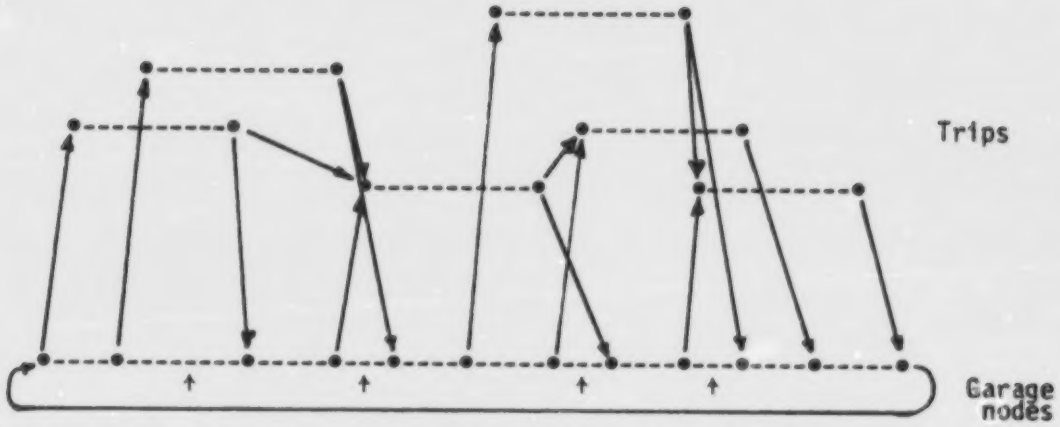
We can state the overall objective of the node-aggregation procedure as one of maintaining flow in a left-to-right direction in Figure 2.2, except on the single return arc. We want to impede any other flow from right to left, because such flow would indicate incongruous trip hooks such as a hook from a trip to itself.

The full node aggregation algorithm is presented in Table 2.1. We have placed vertical arrows in Figure 2.2 at those places where the node groups are formed when applying this algorithm.

We can formally state the Desrosiers-Soumis-Desrochers procedure. Let the series of binary zero-one variables x_{ij} for $(i,j) \in H$ be defined as in VSP1, bearing in mind the new inclusion rule for membership in H. We

Figure 2.2

Schematic diagram of a VSP2
solution graph



Note: Adapted from Desrosiers, Soumis, and Desrochers (1982).

Table 2.1

Node aggregation procedure in VSP2

1. Form a node group consisting of node G_1 , which is always the tail of a pull-out arc. Call this node group the "present group". Let $k = 1$.
2. Increment k by 1 and consider node G_k . If G_k is at the head of a pull-in arc, check to see if there are any tails of pull-out arcs in the present group.
3. If so, close the present group and form a new group consisting of node G_k . This new group then becomes the "present group". If not, add node G_k to the present group.
4. If $k = 2N$ continue to 5. If not, go to 2.
5. Let K' be the number of groups of nodes. Substitute a new node $G^*_{k'}$ for all the nodes in group k' , for $k' = 1,\ldots,K'$. Each arc which had as its tail a node in group k' now has as its tail node $G^*_{k'}$. Similarly, each arc which had as its head a node in group k' now has as its head node $G^*_{k'}$. Place an arc from node $G^*_{k'}$ to node $G^*_{k''}$, where $k'' = k' + 1$, each with a cost of zero and a nominal capacity of N . Place an arc from node $G^*_{K'}$ to node G^*_1 , with a cost of L and a nominal capacity of N .

then define some new notation to handle the aggregated garage nodes. For $i \in N$, let $i'(i)$ be equal to that index k' such that the garage node G_k, where $e_k = g_i$, is in node group k'. Similarly, let $j'(j)$ be that index k' such that the garage node G_k, where $e_k = h_j$, is in node group k'. In other words, $i'(i)$ gives the index of the node group for the node which, in the disaggregated procedure, is at the tail of the (unique) pull-out arc to trip i, and $j'(j)$ similarly gives the node group index for the node at the head of the pull-in arc from trip j. Using this notation, we define the binary zero-one variables $x_{ij'}$ for $i \in N$ as equal to one if trip i is the last trip in a block and zero otherwise, and $x_{j'j}$ for $j \in N$ as equal to one if trip j is the first trip in a block and zero otherwise. We also define a series of nonnegative integer variables $x_{k'}$, $k' = 1,\ldots,K'$ which serve to define flow between the garage nodes. We arbitrarily set $x_{K'}$ as the indicator of flow on the return arc from $G^*_{K'}$ to G^*_1. We then pose the problem

$$\text{VSP2:} \quad \min \sum_{(i,j)\in H} c_{ij}x_{ij} + \sum_{k'=1}^{K'} \sum_{i\in N} c_{iG}x_{ik'} + \sum_{k'=1}^{K'} \sum_{i\in N} c_{Gj}x_{k'j} + Lx_{K'} \tag{2.17}$$

$$\text{Subject to} \quad \sum_{j:(i,j)\in H} x_{ij} + x_{ij'} = 1 \quad , \quad i \in N \tag{2.18}$$

$$\sum_{i:(i,j)\in H} x_{ij} + x_{j'j} = 1 \quad , \quad j \in N \tag{2.19}$$

$$\sum_{i:i'(i)=k'} x_{ii'} - \sum_{j:j'(j)=1} x_{j'j}$$

$$+ (x_{k'-1} - x_{k'}) = 0 \quad , \quad k' = 2,\ldots,K'-1 \qquad (2.20)$$

$$\sum_{j:j'(j)=1} x_{j'j} + (x_{K'} - x_1) = 0 \qquad (2.21)$$

$$\sum_{i:i'(i)=1} x_{ii'} + (x_{K'-1} - x_{K'}) = 0 \qquad (2.22)$$

$0 \leq x_{ij} \leq 1$ and integer for $i \in H$

$0 \leq x_{ii'} \leq 1$ and integer for $i \in N$

$0 \leq x_{j'j} \leq 1$ and integer for $j \in N$

$0 \leq x_{k'} \leq N$ and integer for $k' = 1,\ldots,K'$.

Note that only $x_{K'}$ enters into the objective function (2.17) because the costs on all other arcs between garage nodes are zero. The constraints (2.18) and (2.19) guarantee, respectively, that each trip is either hooked to another trip or is pulled in to the garage, and is either hooked from another trip or is pulled out from the garage. The constraints (2.20), (2.21), and (2.22) guarantee conservation of flow on the garage nodes $G^*_{k'}$.

VSP2 solves the "long arc" problem posed by Bodin et al (1983) in the context of VSP1. A "long arc" is a hook between two trips, the first of which occurs immediately before a period spent in the garage and the second of which occurs immediately afterward. Bodin et al (1983) consider that the simple VSP is not completely specified unless all such "long arcs" are included but recognize that there would be an impossibly large number of them in any reasonable vehicle scheduling problem. As such they counsel the use of VSP1 with

certain precautions. In fact, the problem can be well specified by pooling certain arc flows and ensuring flow conservation on certain nodes, as is done in VSP2.

To illustrate VSP2 and in fact the entire methodology of this research we introduce a fictitious transit trip timetable, which is presented in Table 2.2. This timetable gives the transit service between the community of Apple Grove and its central city of Centreville. The running time for a bus between the two line termini in either direction is 30 minutes. A bus garage is located midway between the two termini and can be accessed out of service from either terminus in ten minutes. In all, there are 65 trips in the Apple Grove timetable.

We assume that deadheading is not permitted between transit trips and that all trip hooks must take place where two trips share a common terminus. We also value L, the constant incremental cost of an additional bus, at 10,000 times T, the incremental cost of a minute of platform time. We also ignore the cost of distance travelled; thus we set the constant K equal to zero.

Table 2.3 presents a summary of the solution to VSP2 applied to the Apple Grove scheduling example. We see that the garage node aggregation procedure of Table 2.1 has reduced the problem size substantially. We also note that there are nine blocks in the vehicle solution: three which operate all day long, three which operate only in the morning, and three which operate only in the evening. Thus the total required fleet size is six vehicles.

A FORTRAN program to set up and solve VSP2 using RNET (Grigoriadis and Hsu, 1979) is included in this dissertation as Appendix C.

Table 2.2

Trip timetable for the
Apple Grove scheduling example

Leave Apple Grove	Arrive Centreville	Leave Centreville	Arrive Apple Grove
7:45 a.m.	8:15 a.m.	8:00 a.m.	8:30 a.m.
8:00	8:30	8:15	8:45
8:10	8:40	8:30	9:00
8:20	8:50	8:40	9:10
8:30	9:00	8:50	9:20
8:40	9:10	9:00	9:30
8:50	9:20	9:15	9:45
9:00	9:30	9:30	10:00
9:15	9:45	9:50	10:15
9:30	10:00	10:00	10:30
9:45	10:15	10:15	10:45
10:00	10:30	10:30	11:00
10:20	10:50	10:50	11:20
10:40	11:10	11:10	11:40
11:00	11:30	11:30	12:00 p.m.
11:20	11:50	11:50	12:20
11:40	12:10 p.m.	12:10 p.m.	12:40
12:00 p.m.	12:30	12:25	12:55
12:20	12:50	12:40	12:10
12:40	1:10	12:55	1:25
12:55	1:25	1:10	1:40
1:10	1:40	1:20	1:50
1:25	1:55	1:30	2:00
1:40	2:10	1:40	2:10
1:50	2:20	1:50	2:20
2:00	2:30	2:00	2:30
2:10	2:40	2:10	2:40
2:20	2:50	2:20	2:50
2:30	3:00	2:30	3:00
2:40	3:10	2:40	3:10
2:50	3:20	2:50	3:20
3:00	3:30	3:00	3:30
		3:20	3:50

Table 2.3

VSP2 solution summary for the
Apple Grove scheduling example

Block	Garage	Start Time/Loc	End Time/Loc	Garage	Time
1	(7:35)	7:45/AG	9:20/CV	(9:30)	1:55
2	(7:50)	8:00/AG	15:10/AG	(15:20)	7:30
3	(7:50)	8:00/CV	15:10/CV	(15:20)	7:30
4	(8:00)	8:10/AG	15:30/AG	(15:40)	7:40
5	(8:10)	8:20/AG	9:20/AG	(9:30)	1:20
6	(8:30)	8:40/AG	10:45/AG	(10:55)	2:25
7	(12:15)	12:25/CV	15:30/CV	(15:40)	3:25
8	(13:10)	13:20/CV	15:50/AG	(16:00)	2:50
9	(13:40)	13:50/CV	15.20/AG	(15:30)	1:50
			Total vehicle time		31:45

Total nodes

Without garage aggregation	260
With garage aggregation	165

Total arcs

Without garage aggregation	387
With garage aggregation	292

Objective function at optimum 60235

2.2.3 Other vehicle scheduling methods

2.2.3.1 The classic minimum fleet size problem. No discussion of vehicle scheduling would be complete without mention of the classic minimum fleet size problem (Dantzig and Fulkerson, 1954). As in VSP1 and VSP2, we want a set of N trips to be served by a fleet of vehicles, and some one of these vehicles is to serve each trip once and only once. Now, however, we are only interested in minimizing the fleet size, and have no secondary objectives such as those of minimum vehicle time or vehicle travel distance.

Let V_i , $i \in N$ be a set of nodes associated with the trips to be scheduled. If we propose a model by which feasible flow from V_i to V_j implies that trip j follows trip i in a vehicle schedule, as we have done in VSP1 and VSP2, then we can choose an ordering of the nodes V_i which forms an acyclic directed graph, that is, an ordering whereby an arc from V_i to V_j implies that $j > i$. The assignment of indices i to trips in increasing order of arrival time u_i clearly satisfies this criterion, since $i < j$ would then imply $u_i \leq u_j$. Since $t_i < u_i$ for any i , there could clearly be no feasible flow from V_j to V_i . We let H be the set of all pairs (i,j) where

$$t_j > u_i + h(i,j)$$

that is, the set of pairs (i,j) where trip j could feasibly follow trip i in a vehicle schedule. We place no upper bound on the difference

$$t_j - u_i + h(i,j) .$$

We define a "path" as a sequence of arcs

$$(i_1,i_2),\ (i_2,i_3),\ \ldots,\ (i_{n-1},i_n)\ \varepsilon\ H$$

where (i,j) indicates an arc from V_i to V_j and $n < N$. The problem of minimum fleet size can be posed as the problem of finding the smallest number of paths which cover all the nodes V_i. Dilworth (1950) proved a theorem which implies that the problem of finding the minimum number of paths is the same as that of finding the maximum number of pairs (i,j) such that there is no path from V_i to V_j, that is, a pair of trips i and j which cannot be served by the same vehicle. Following Ford and Fulkerson (1962), let x_{ij} for $(i,j)\ \varepsilon\ H$ be a series of binary zero-one variables such that

$$x_{ij} = \begin{cases} 1 \text{ if trip j follows trip i in a vehicle schedule} \\ 0 \text{ otherwise} \end{cases}$$

We then pose the problem

$$\max \sum_{(i,j)\varepsilon H} x_{ij}$$

$$\text{Subject to } \sum_{j\varepsilon N} x_{ij} \leq 1\ ,\ i\ \varepsilon\ N$$

$$\sum_{i\varepsilon N} x_{ij} \leq 1\ ,\ j\ \varepsilon\ N$$

$$0 \leq x_{ij} \leq 1 \text{ and integer for } (i,j)\ \varepsilon\ H$$

The quantity

$$N - \sum_{(i,j)\varepsilon H} x_{ij}$$

then gives the required minimum number of vehicles.

Ford and Fulkerson (1962) also give a simple algorithm, called the Dilworth chain decomposition, for solving this problem. This algorithm could be adapted to the present concepts and notation as follows. Choose a trip j such that there is no $i \in N$ for which $(i,j) \in H$. Find $k \in N$ such that $(j,k) \in H$ and $t_k - u_j + d(j,k)$ is minimal for all such k. Hook trip k to trip j, and repeat the same process with trip k. Continue in this fashion until a trip $s \in N$ is reached for which there is no trip $t \in N$ such that $(s,t) \in H$. Then remove the entire set of trips $\{ j,k,\ldots,s \}$, where $(j,k), \ldots, (r,s)$ is the path formed from the above procedure, from the problem and continue.

Note, however, that there is no mechanism in this algorithm to take into account decreasing service demand in the middle of the day and thus nothing to prevent solutions which involve very large layovers, in the manner of VSP1. Despite this limitation, the Dilworth chain procedure has been used to simulate transit vehicle scheduling in the UCOST transit cost estimation component of the Urban Transportation Planning System package, which is provided to local transportation planning agencies by the United States Department of Transportation (Bodin and Rosenfield, 1976; Bodin, Rosenfield, and Kydes, 1981). Importantly, UCOST uses a two-stage procedure where during the first stage the set of possible hooks H (using our notation) is limited by a maximum layover parameter (Bodin et al, 1981). The sequences of trips which have been aggregated into chains according to this first stage are then used as nodes in a second application of the algorithm which does not make use of such a maximum layover parameter. The reason given by these authors for the use of this two-step procedure is not the elimination of superfluous layover time but the reduction of the problem to manageable size

because of a much smaller arc set at each of the two stages.

2.2.3.2. An approach using "deficit functions". The optimizing methods discussed thus far in this chapter have been devised in order to take account of the large number of trip hooking possibilities in vehicle scheduling applications and very few non-optimizing techniques exist this address this problem in a systematic way. One exception is an approach to scheduling which uses "deficit functions" (Gertsbach and Gurevich, 1977; Stern and Ceder, 1981; Ceder and Stern, 1983). A deficit function is a step function defined over time at a specific trip terminus which increases by one when a bus departs from the terminus and decreases by one when a bus arrives at the terminus. Thus the value of the deficit function for terminus k at time t gives the number of vehicles accounted for by trips leaving k up to and including time t. The maximal value, over t, of the deficit function at k is the contribution to total vehicle requirements made by trips leaving k which cannot be satisfied by trips leaving other termini and arriving at k. The sum, over termini, of all such maximal values gives the total fleet size requirements (Gertsbach and Gurevich, 1977).

The scheduling procedure using deficit functions is a computer-assisted manual technique. The method is one of manually adding deadhead trips and possibly adjusting trip times in order to reduce the maximum deficit at certain termini without increasing the maximum deficit throughout the rest of the system (Stern and Ceder, 1981; Ceder and Stern, 1983).

An interesting aspect of deficit functions (Stern and Ceder, 1981) is that they give upper and lower bounds on fleet size requirements for a particular schedule. If no deadhead trips at all are inserted the total over all termini of maximum deficits will give the upper bound on fleet size. On

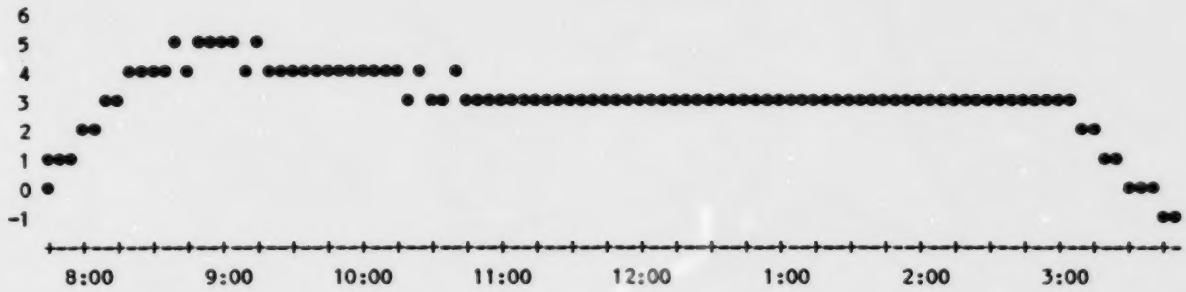
the other hand, the maximum value of the "global" deficit function, which is the vertical sum over all termini of complete deficit functions, gives a lower bound on the fleet size. The global deficit function is equivalent to a histogram showing the number of trips in simultaneous operation at any time of the day. The fleet size cannot drop below the maximum number of trips in simultaneous operation at any one time.

Figure 2.3 shows the deficit functions for each terminus in the Apple Grove problem: Figure 2.3(a) for the Apple Grove terminus, Figure 2.3(b) for the downtown Centreville terminus. The sum of the maximum values of these two functions is six. Figure 2.3(c) gives the "global" deficit function for the complete Apple Grove problem and the lower bound on fleet size is the maximum value of this function, or six. Thus we cannot save vehicles through deadheading in the Apple Grove example, as would perhaps be expected in a simple problem involving only two termini.

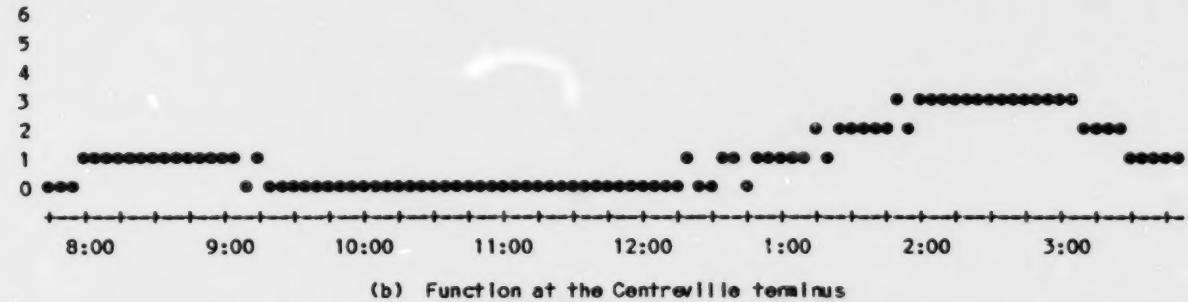
2.2.4.3 A vehicle scheduling method which incorporates crew scheduling constraints. Ball, Bodin, and Dial (1983) have devised a radical reformulation of the joint vehicle/crew scheduling problem which introduces the notion of a d-trip. A d-trip is any portion of a transit trip in which the vehicle and its operator must stay together. For example, if a trip begins at terminal A and goes to terminal B passing through R, where R is a relief point where the driver can be relieved and replaced by another operator, we have two d-trips: one from A to R and one from R to B. Ball et al see the d-trip as the basic building block of transit scheduling, in that vehicle schedules can be built up by joining d-trips together in one way (if relief points are ignored, the problem reduces to the simple VSP) and crew schedules can be built up by joining d-trips together in another way (if

Figure 2.3

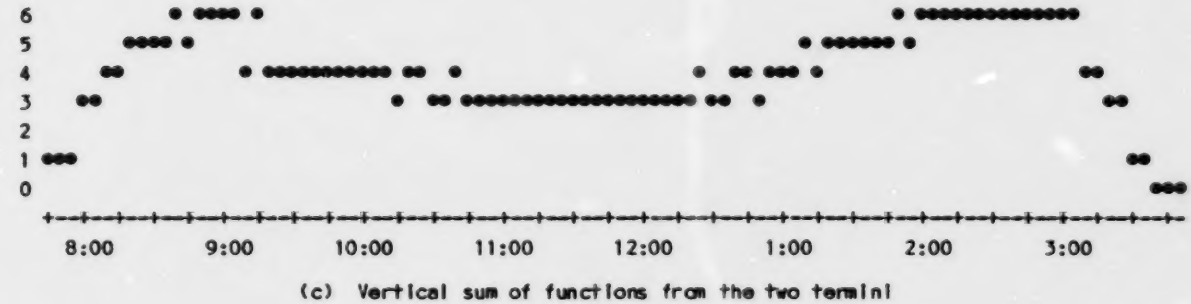
Deficit functions for the
Apple Grove scheduling example



(a) Function at the Apple Grove terminus



(b) Function at the Centreville terminus



(c) Vertical sum of functions from the two termini

Note: The vertical axis on all graphs is number of vehicles.

trip hooks are assumed given, the problem reduces to the crew scheduling problem).

Ball et al (1983) formulate the general joint vehicle/crew scheduling problem as a pair of partitioning problems on acyclic graphs. The two graphs share a set of nodes (the d-trips) but each has quite a different set of arcs, owing to the fact that the two problems to be solved are very different. The arc set of the crew scheduling graph is by far the more complicated, because of a more complex set of possibilities in the crew scheduling problem. The objective function in the crew scheduling graph is nonlinear, being drawn from the collective agreement in force where the scheduling is being done. Finally, there are constraints introduced between the two problems to guarantee that the two solutions will be consistent with one another.

As is true with all exact formulations of the crew scheduling problem, even those which assume vehicle schedules to be fixed in advance, the number of variables quickly grows too large to be manageable. In particular, Ball et al note that the number of nodes (d-trips) in a problem of reasonable size might be as large as 6,000 and that the number of arcs can be expected to be a constant times the square of this. To be solvable, therefore, the problem must once again be decomposed.

What is interesting from the point of view of vehicle scheduling is the way Ball et al (1983) carry out this decomposition. The method builds up pieces of work for operators by joining together d-trips in the data base according to a heuristic. Actual operator duties are only formed after these pieces of work are already available, by solving a matching problem which incorporates actual manpower cost in the objective function.

Ball et al (1983) have in fact changed the normal order of data flow in transit scheduling. Because of the primacy of pieces of work to the method,

vehicle schedules are created only as a by-product of the manpower scheduling process. To see this, note that pieces of work are always served by the same driver and the same vehicle. Furthermore, pieces of work always begin and end at either a pull-out, a pull-in, or a relief point. A relief point always occurs at some point during a trip, which must be served by a single vehicle. Thus once pieces of work have been formed by joining together d-trips, the set of vehicle schedules is known. This stands in constrast to the traditional methods described in Chapter 1 where vehicle schedules are created explicitly before the manpower scheduling process even begins.

Importantly, however, both the decomposition of Ball et al (1983) and the traditional decomposition share a common characteristic: The cost of a manpower solution, which depends in fact on how pieces of work are matched into full crews, is not known until after the vehicle schedule is already fixed. Thus these authors propose to replace the traditional problem decomposition by another one which still does not fully breach the information gap between vehicle and manpower scheduling solutions. Ball et al (1983) provide some numerical results to support the efficacy of their technique, but additional experience with this method will be needed before the full range of its utility is understood.

2.3 Transit service costing methods

McGillivray, Kemp, and Beesley (1980) review procedures for estimating costs in transit and they stress that an important aspect of any cost estimation method is a clear idea of what information the method is supposed to provide and of what decision will be made as a consequence of having this information. Accordingly, they divide available approaches into three categories: those used to estimate the cost impact of changes to transit

service, those used to evaluate changes in transit production possibilities (including scheduling constraints and remuneration policies), and those used to allocate portions of total cost to different jurisdictions. This review concentrates on the first two of these types of cost estimation procedures, as the method of this research shares their objectives more than that of the third.

2.3.1 Procedures for estimating the cost of service changes. The most primitive cost estimation procedure is simple average cost (McGillivray et al, 1980). For example, a "cost per bus hour" could be computed as the total operating budget of the transit property divided by the number of vehicle hours it operates. To estimate the cost of a proposed service change one simply multiplies the change in the number of vehicle hours by this average cost figure. As the number of vehicle hours implied by a particular trip schedule is relatively easy to predict, especially in the absence of any interlining between trip termini, this model can yield a cost estimate quickly. Practitioners have realized, however, (McGillivray et al, 1980), that the nonlinearity of the relationship between vehicle time and total labor cost makes such a method crude at best.

A class of more complex procedures, still largely within the average cost framework, could be called "account allocation" models. Such models partition the total operating budget into components which are then allocated to measurable "factors" or surrogates for causes of fixed and avoidable cost. Each component is divided by the current actual occurrance of each of the factors to yield an average cost per unit of the factor. To estimate the cost of a proposed service change one measures the anticipated changes in the level of each of these factors and then creating a weighted sum of these anticipated

changes and the computed average costs. A two-factor model which partitions the budget into accounts allocatable to vehicle miles and vehicle hours is that of Ferreri (1969). A typical three-factor model would use vehicle miles, vehicle hours, and total fleet size (Cervaro, Wachs, Berlin, and Gephart, 1980). One four-factor model includes additionally a factor for the number of operators (Ferreri, 1969). The UCOST subsystem of the Urban Transportation Planning System (Bodin, Rosenfield, and Kydes, 1981) partitions operating cost into no fewer than fifteen separate components. One method (Arthur Andersen and Company, 1974) creates a three-by-three table of accounts which crosses vehicle miles, vehicle hours, and fleet size with variable, semi-variable, and fixed costs.

These approaches have in common that average cost figures remain constant regardless of the service change to be evaluated. But practitioners have realized that average cost per bus hour during service peaks will probably not be the same as this average cost outside of the service peaks (McGillivray et al, 1980). Thus some methods have appeared in which average cost per vehicle hour is not assumed to be a constant. Arthur Andersen and Company (1974) has devised a method whereby a property estimates the average cost of service per peak and off-peak vehicle hour using linear regression. In this method, the service day is arbitrarily partitioned into peak and off-peak periods. Then the statistics

$$P = \frac{\text{peak bus minutes}}{\text{total bus minutes}}$$

and

$$C = \frac{\text{crew minutes paid}}{\text{total bus minutes}} \qquad (2.20)$$

are computed for each of the manpower assignments in actual operation at the property. The linear regression relating C to P is computed and the regression line is extrapolated to $P = 0$ and $P = 1$. The value of the regression at $P = 1$ is taken to be the average cost per peak hour and this value at $P = 0$ is taken to be the average cost per off-peak hour. In other words, a linear extrapolation from existing data is used to estimate the labor cost per unit of vehicle time for a hypothetical duty which involved no peak working and to estimate the same quantity for another hypothetical duty which involved nothing but peak working.

Cherwony and Mundle (1978) separately estimate average cost per peak and off-peak vehicle hour as a linear function of what they call "labor productivity" in the peak and in the off-peak and of the ratio of peak to off-peak vehicle hours. Their "labor productivity" quantity is the same ratio as (2.20). Note that they treat this ratio as an independent variable in their analysis, while the Arthur Andersen method treats it as a dependent variable which can presumably be explained by the percentage of time worked during the peak by operators.

Reilly (1977) computes average cost per peak and off-peak vehicle hour from existing manpower assignments using the amount of straight time, or time paid involving no premiums at all, and overtime in the peak and off-peak periods in these assignments and by adding a factor which is intended to reflect capital costs. Reilly's model does not take into account other guarantees and premiums which apply to manpower scheduling.

Note that the definition of the "peak" in each of these approaches is totally arbitrary, even though the cost estimates they provide may be sensitive to the defined limits of "peak" and "off-peak" service. Some

researchers (McClenahan, Nicholls, Elms, and Bly, 1978) have tried to adopt a standard definition of the "peak" based on stated objectives of the transit service. But there does not seem to be universal agreement on how to define the "peak" (McGillivray, Kemp, and Beesley, 1980).

In addition, the eight companies to which McClenahan et al (1978) applied the Arthur Andersen method show very different peak and off-peak average costs. The peak-hour average cost coefficients presented in their study vary by a factor of two, and the off-peak average cost coefficients vary by a factor of six. The variation arises, presumably, from the nature of the schedules themselves and from manpower scheduling constraints in force at the various properties under study. But the method does not probe for the causes of this variability, even though the cost coefficients may be sensitive to the very schedule changes which are motivating the cost estimation.

The London (England) Transport Executive (Wagon and Baggeley, 1975) reports an average cost method which involves finding average cost per type of duty -- one-piece or two-piece -- required to cover a particular schedule. The total number of duties T required to cover a proposed schedule is found by dividing the number of vehicle hours in the new schedule by the average number of vehicle hours per shift currently operating in the property as a whole, and then rounding to an integer. They define a quantity P as the number of "peak ends", or the number of vehicles required to serve just the morning peak plus the number required to serve just the evening peak. In the Apple Grove scheduling example, the VSP2 solution of Table 2.2 shows three morning peak vehicles and three evening peak vehicles. Thus the value of P in this example would be 6. The London Transport model assumes that any one-piece duty can cover at most one of the peaks, and that any two-piece duty can cover both peaks. Thus the number of one-piece duties, S, plus twice

the number of two-piece duties, W, will equal P, and we have the two equations

$$S + 2W = P$$

$$S + W = T$$

which can be solved for S and W:

$$W = P - T$$

$$S = 2T - P$$

Finally, the average cost in operator pay per one-piece duty and per two-piece duty is calculated for the existing manpower schedule at the property and the total cost of the new service is estimated as a weighted sum of W and S. London Transport has found (Wagon and Baggeley, 1975) that such a method will provide reasonable cost estimates under a variety of scheduling conditions.

Finally, we note that with the advent of computerized transit data management and scheduling tools (Wren, 1981) much of the job of estimating the cost of proposed service changes can be turned over to the computer. Some packages, notably the HASTUS-Macro package (Blais and Rousseau, 1982) which is described in the next section and in Chapter 3, have been explictly designed as estimation tools. Others were created with actual vehicle and manpower scheduling in mind, but once calibrated to a particular property's needs there is nothing to prevent their use to estimate the cost of serving proposed new schedules. RUCUS and RUCUS-II, in particular, have been applied to just this sort of cost estimation (Luedtke, 1983).

2.3.2 Methods for estimating the cost of changes in manpower scheduling conditions. In contrast to the methods for estimating the cost of changes in transit service, there are no recent manual techniques for estimating the cost of changes in manpower scheduling constraints or remuneration policies. The HASTUS-Macro software system, however, was devised for just this application (Blais and Rousseau, 1982). While the mathematical model used by this set of programs is fully discussed in Chapter 3, we will briefly summarize some of the features of this program in this section.

HASTUS-Macro accepts a set of commands which are intended to give the values of certain parameters and direct the operation of the program. These parameters are in fact values of the limits, premiums, and guarantees described in Table 1.1. Using these parameter values and rudimentary vehicle schedule information HASTUS-Macro constructs a large linear program where the primal variables represent different kinds of operator duties and the primal constraints assure that, in aggregate, there are enough duties scheduled to satisfy service requirements, certain system-wide scheduling restrictions, and certain feasibility requirements imposed by the given vehicle solution. In other words, HASTUS-Macro relaxes the manpower scheduling problem and uses the relaxed problem to estimate the cost of manpower to serve a particular vehicle solution assuming certain values of its parameters. In general, the solution to HASTUS-Macro will not be integer. Since the object of the exercise is cost estimation and not actual manpower scheduling, however, this nonintegrality is not seen as a major shortcoming.

Blais and Rousseau (1982) report some results of applications of HASTUS-Macro in actual cost estimation situations. Dupuis (1983) and Mitchell (1983) report additional North American experience and Garnier (1983) reports some European experience.

Although HASTUS-Macro is the only computer package designed explicitly for cost estimation, other manpower-scheduling packages can be used for this purpose as well. Chomitz and Lave (1981), for example, report a study in which they use a version of RUCUS (Roberts, 1973) to estimate the impact on total labor cost of changes in the spread limit and in the limit on the allowable percentage of part-time operators at five real but unnamed American transit properties. In principle most any manpower scheduling tool could be used to do the same thing, although some would have fewer hard-coded scheduling parameters than others and hence would be easier to use to simulate the effects of parameter changes. For an overview of different available packages for manpower scheduling see Wren (1981) and Rousseau (1983).

Chapter 3

A model for approximating optimal transit vehicle schedules

3.1 General introduction

This chapter and the next two present the approach to transit vehicle scheduling developed in this research. The new vehicle scheduling methods is similar in pholosophy to that of the HASTUS manpower scheduling program (Rousseau, Lessard, and Blais, 1983) which finds an optimal solution to a linear programming relaxation of the original manpower-scheduling problem and then proceeds to a feasible solution using information available from the relaxation. The present approach shares with HASTUS the use of a linear programming relaxation, now applied to the task of finding transit vehicle schedules which are efficient in their use of manpower.

The method is, in outline, as follows:

1. An initial vehicle schedule solution is found using a network flow model which minimizes a composite objective function. The transhipment model VSP2 which was introduced in Section 2.2.2 serves as the principle analytic tool in this step.
2. Using information from the initial solution and manpower scheduling parameters appropriate to a particular scheduling situation, a large linear program is set up and solved. The initial solution serves to identify the levels of transit service which must be respected during the course of the scheduling method and also to restrict the size of the linear program by identifying certain variables which would not

likely be found in an optimal solution. The linear model is described abstractly in Chapter 3 and an example of it using the Apple Grove timetable data is presented in Chapter 4.

3. The initial vehicle schedule solution is then improved marginally using information from the linear model. We develop, in Chapter 5, a heuristic designed to identify those vehicle schedules in the initial solution which can never cost out optimally and to eliminate them from the solution.

This chapter presents the basic principles of the linear programming approximation model. As this model is a straightforward extension of the linear programming relaxation used in HASTUS, the first part of the chapter is devoted to a discussion of that relaxation. The second part of the chapter describes the linear model used in this research.

3.2 The HASTUS-Macro crew schedule estimation program

HASTUS-Macro is a software package designed to estimate manpower cost in urban transit. In addition, HASTUS-Macro forms part of a suite of programs, the HASTUS system, which is a commercially available vehicle and manpower scheduling software package (Blais and Rousseau, 1983; Rousseau, Lessard, and Blais, 1983). HASTUS does manpower scheduling in two major phases. A first phase solves a relaxation of a manpower scheduling problem, which is set up as a large linear program. A second phase uses the results of this linear program to establish a real manpower schedule close to the optimum of the relaxed problem.

This section is only concerned with the linear program used in HASTUS. Full details of the entire HASTUS system are available elsewhere (Blais and

Rousseau, 1983; Rousseau et al, 1983) and will not be given here. In addition, we will give a name, HM1, to this linear program to emphasize the distinction between the mathematical model itself and the commercial software package which exploits it.

The HM1 linear program is as follows (Blais and Rousseau, 1982). Consider transit service at a particular transit property on a particular day and assume that vehicle schedules are already available. If we divide this day into periods $p = 1,\ldots,P$, we can calculate service demands

$$b_1, b_2, \ldots, b_p$$

representing the maximum level of service implied by the vehicle schedules for each period. The length of these periods may vary from problem to problem but is constant for any one problem. Thus P depends on the length of service and the chosen period length. We then consider a set $\mathbf{D}$ of 4-tuples (ij,mn) where either

$$i,j,m,n \in P = \{1,\ldots,P\}, \; i \leq j, \text{ and } m \leq n \text{ and } m > 0,$$

or

$$i,j \in P, \; i \leq j, \text{ and } m = n = 0.$$

We then define a variable $x_{ij,mn}$ associated with each element of $\mathbf{D}$. An increase in $x_{ij,mn}$ by one unit represents one person working from hour i to hour j inclusive, and also from hour m to hour n inclusive (in a two-piece duty) if $0 < m \leq n$. For simplicity, this presentation considers only one- and two-piece duties; the development can be extended without difficulty to duties with more pieces.

From the terms of the collective agreement in force at a particular

transit property a number $c_{ij,mn}$ can be calculated for each (ij,mn) in D. This number represents the cost the transit property would incur if it were to employ one person to carry out the work schedule implied by $x_{ij,mn}$. Chapter 1 described how the rules which regulate manpower usage in transit are complex and vary from property to property. The manpower scheduling problem in transit is not fully specified, however, until these work rules are known. Thus the HM1 linear program would almost never be the same for manpower schedule cost estimation at two different properties.

If we allow each $x_{ij,mn}$ to take on nonnegative, not necessarily integral values, the linear function

$$\sum_{(ij,mn)\in D} c_{ij,mn} x_{ij,mn} \tag{3.1}$$

gives an estimate of the total operator cost at the transit property in question. HM1 seeks to minimize (3.1) subject to certain constraints. First, there are naturally "service-covering" constraints which assure that there will be as many operators on duty at a particular period p as there are vehicles in service at that period. If we define a set

$$D_p = \{ (ij,mn) : \ i \leq p \leq j \quad \text{or} \quad m \leq p \leq n \}$$

we require that

$$\sum_{(ij,mn)\in D_p} x_{ij,mn} \geq b_p \ , \quad p \in P \tag{3.2}$$

where b_p is the service requirement at time p.

Second, consider a constant W which is equal to the maximum number of periods in a "small block", that is, a vehicle schedule which the analyst considers to be too short to be cut when forming pieces of work. Such a

decision might arise, for example, from a piece guarantee provision in the union contract. For example, if any piece of work is paid using a minimum guarantee of two hours, then any vehicle schedule of less than four hours in length should not be cut in forming a manpower schedule. This is because it would be impossible to cut into two pieces a block of less than four hours without creating at least one piece of less than two hours, which would incur a piece guarantee premium. In this example, then, we would choose W to be the maximum integer such that W periods amounted to less than 4 hours.

Now consider the set S of pairs (qr), where $q,r \in P$, $q < r$ and $(r - q + 1) < W$. We associate with each element $(qr) \in S$ a subset D_{qr} of D, defined as

$$D_{qr} = \{ (ij,mn) \in D : (ij) = (qr) \text{ or } (mn) = (qr) \}$$

The "small block" constraints

$$\sum_{(ij,mn) \in D_{qr}} x_{ij,mn} \geq v_{qr}, \quad (qr) \in S \tag{3.3}$$

where v_{qr} is the number of blocks from q to r inclusive in the given set of vehicle schedules, indicate that the manpower estimate must allow for the existence of these small blocks. Note that in any given HM1 problem most of these v_{qr} will be zero, since they measure the <u>actual</u> occurrance of particular blocks in a given vehicle schedule solution.

Third, associate with each element p of P a subset $D_{p'}$ of D defined as

$$D_{p'} = \{ (ij,mn) \in D : i = p \text{ or } m = p \}$$

"Pull-out" constraints of the form

$$\sum_{(ij,mn)\in D_p'} x_{ij,mn} \geq v_p \quad , \quad p \in P \tag{3.4}$$

where v_p is the number of blocks which begin at time p in the given set of vehicle schedules, guarantee that the given vehicle block start times are respected in the manpower schedule estimation. The "short block" and "pull-out" constraints are described by Rousseau and Blais (1982) as "feasibility constraints" because they serve to direct the linear program toward a solution which is more easily converted into feasible duties by HASTUS-Micro.

Finally, there are F supplemental constraints, usually property-specific, of the form

$$\sum_{(ij,mn)\in C_f} x_{ij,mn} \geq q_f \quad , \quad f = 1,\ldots,F \tag{3.5}$$

where C_f is some subset of D, and q_f is some number, for $f = 1,\ldots,F$. Such a constraint might specify, for example, that n percent of all duties must contain only one piece, where n is some number.

In principle, D could be composed of all possible (ij,mn). In practice, however, it has proven satisfactory (Blais and Rousseau, 1982) to limit the size of D by applying certain preselection rules. For example, it is possible to specify that at least one of the pieces of a two-piece duty must cover one of the service peaks.

It has also been found in practice that slack in the block-covering constraints (3.2) is slight or inexistant (Blais, 1984). What this means is that certain of the "pull-out" constraints are effectively redundant. To see this, consider a particular vehicle schedule solution and let K be the subset of P consisting of all periods in which at least one vehicle schedule begins in this solution. Similarly, let L be the subset of

P consisting of all periods in which at least one vehicle schedule ends. Now consider a period $q \in K$ and assume, for the moment, that there is no period $r \in L$ such that $q = r + 1$. In other words, q is such that at least one vehicle schedule begins during q and no vehicle schedule ends immediately before q. Note that this assumption implies that $b_q > b_{q-1}$ and that $b_q - b_{q-1}$ is the number of pull-outs at period q, or v_q. From (3.2) we have that

$$\sum_{(ij,mn)\in D_q} x_{ij,mn} = b_q + \varepsilon_q \tag{3.6}$$

and also that

$$\sum_{(ij,mn)\in D_{q-1}} x_{ij,mn} = b_{q-1} + \varepsilon_{q-1} \tag{3.7}$$

where ε_q and ε_{q-1} are slack variables. Subtracting (3.6) from (3.7) yields

$$\sum_{(ij,mn)\in D_q} x_{ij,mn} - \sum_{(ij,mn)\in D_{q-1}} x_{ij,mn} = (b_q - b_{q-1}) + (\varepsilon_q - \varepsilon_{q-1}) \tag{3.8}$$

Our assumption about the period q, however, implies not only that

$$(b_q - b_{q-1}) = v_q$$

but that the left-hand side of (3.8) is equal to that of (3.4). Thus if we assume $(\varepsilon_q - \varepsilon_{q-1})$ approximately equal to zero we have that

$$\sum_{(ij,mn)\in D_q} x_{ij,mn} \approx v_q$$

which is in fact nearly equivalent to the q^{th} pull-out constraint (3.4).

If in our vehicle schedule solution there is some period $q \in K$ for

which there is also a period $r \in L$ such that $q = r + 1$, then we can no longer conclude that

$$(b_q - b_r) = v_q$$

and thus that the pull-out constraint is nearly duplicated in the service-covering constraints. It is in such a situation that the HM1 model includes an explicit pull-out constraint for period q.

The second phase of the HASTUS solution process, HASTUS-Micro, proceeds to a feasible solution of the original crew scheduling problem. See Rousseau et al (1983) for details on the creation of actual duties in HASTUS.

3.3 An extended linear program for estimating efficient vehicle schedules

The right-hand side of the HM1 linear program is not constant with respect to alternate sets of vehicle schedules in that the constraints (3.3) and (3.4) depend on block lengths and start times. More specifically, the values of the v_{qr} in (3.3) and of the v_p in (3.4) will be different depending on the vehicle schedules presented to HM1 and, depending on the cost coefficients $c_{ij,mn}$ (which depend on the provisions of a particular collective agreement) the solution to HM1 will vary accordingly. And yet just as the solution to a linear program may depend on its right-hand side, it is reasonable to expect that different sets of vehicle schedules, each of which covers the service histogram, might be served with manpower with differing degrees of efficiency.

The present research attempts to exploit this property of the HM1 model by considering an extension in the following sense. In addition to the set D of 4-tuples representing approximate potential duty types, consider a set B of pairs (qr), $q,r \in P$, corresponding to potential vehicle schedules. For the moment, assume that B contains all possible pairs

(qr) of elements of P such that $q \leq r$. We can associate with each element $(qr) \in B$ a variable y_{qr} of which one unit indicates one vehicle in service from period q to period r inclusive. In the same way that the $x_{ij,mn}$ are only approximations to real duties, the y_{qr} are only abstractions and are not the output from a process of hooking together actual vehicle movements.

We can use the variables y_{qr} to introduce a variable vehicle schedule structure into HM1. Let the subsets B_p of B, for $p \in P$, be defined as follows:

$$B_p = \{ (qr) \in B : q \leq p \leq r \}$$

and define a subset $B_{p'}$ of each B_p as follows:

$$B_{p'} = \{ (qr) \in B_p : q = p \}$$

We pose the following simple relations, making use of the notation in inequalities (3.2), (3.3), and (3.4):

$$b_p = \sum_{(qr) \in B_p} y_{qr} \quad , \quad p \in P \tag{3.9}$$

$$v_{qr} = y_{qr} \quad , \quad (qr) \in S \tag{3.10}$$

$$v_p = \sum_{(qr) \in B_{p'}} y_{qr} \quad , \quad p \in P \tag{3.11}$$

In words, the right-hand side of the HM1 service-covering and pull-out constraints can be constructed as the sum of y_{qr} variables instead of accepted as given constants and that of the small-block constraints can be constructed as another y_{qr} variable. With this in mind we can combine (3.2)

through (3.4) and (3.6) through (3.8) to yield

$$\sum_{(ij,mn)\in D_p} x_{ij,mn} - \sum_{(qr)\in B_p} y_{qr} \geq 0 \quad , \quad p \in P \tag{3.12}$$

$$\sum_{(ij,mn)\in D_{qr}} x_{ij,mn} - y_{qr} \geq 0 \quad , \quad (qr) \in S \tag{3.13}$$

$$\sum_{(ij,mn)\in D_p} x_{ij,mn} - \sum_{(kl)\in B_p} y_{qr} \geq 0 \quad , \quad p \in P \tag{3.14}$$

We note in passing that the set S is actually a subset of B, that is,

$$S = \{ (qr) \in B : (r - q + 1) \leq W \}$$

In other words, S is that subset of candidate vehicle schedules which must maintain their integrity in any manpower scheduling solution. We note also that while a number of the v_{qr} variables are zero in any particular HM1 problem, so that the effective number of constraint rows would be quite small, the general form (3.13) of the small-block constraints requires that there be a constraint row corresponding to each (qr) pair in S.

Service covering constraints enter the new formulation as well, but in a different way. In HM1 the constraints (3.2) require that service requirements arising from a fixed block solution be respected. In the extended linear program, by contrast, (3.2) has in fact been replaced by (3.9), where the blocking scheme is not fixed but variable. The natural form of a new set of service-covering constraints would be

$$\sum_{(qr)\in B_p} y_{qr} \geq s_p \quad , \quad p \in P \tag{3.15}$$

where the s_p are numbers arising somehow from the service requirements

implicit in the trip timetable. For reasons which are discussed in section 3.4 we choose $s_p = b_p$ for all $p \in P$, where as in HM1 each b_p is the height at period p of a demand histogram created from discretizing an initial vehicle schedule solution.

With the objective function to be minimized unchanged at (3.1) and the supplemental constraints (3.5) unchanged as well, we can add nonnegativity requirements on the $x_{ij,mn}$ and the y_{qr} to completely specify a new linear program, which we will call HM2, which generalizes and extends HM1.

3.4 The use of an initial vehicle schedule solution

We make use of an initial vehicle schedule solution, found using the VSP2 model of Section 2.2.2, to set certain parameters of the HM2 model and to restrict its size. Such an initial vehicle solution is used in two ways. First, we construct the sets K and L of start and end periods which occur in that solution. We then restrict the set B by requiring that any pair $(qr) \in B$ be such that

$$q \in K, \quad r \in L, \quad \text{and} \quad q < r.$$

In other words, we only consider potential approximate vehicle schedules which start during the same period as at least one of the VSP2 blocks and end during the same period as at least one of these blocks.

Second, we identify each of the constants s_p in equation (3.13) with the height of the VSP2 service histogram during the p^{th} period, for all p. These values of s_p are identical to the values the b_p would take if we were solving HM1 assuming the VSP2 solution to be fixed.

Such a procedure represents two assumptions. The first recognizes that VSP2 will put vehicles in and out of service in response to demands for such service arising from the trip schedule. We assume that an optimal vehicle solution will necessarily respond to the same demands. The second assumption recognizes that any vehicle schedule solution will include some platform time which corresponds to layovers, deadheads between trips, and deadheads to and from the garage and that we are therefore correct to estimate the s_p slightly larger than those arising from a simple histogram of unhooked trips.

This method for specifying B and the s_p has the property, in fact, that there is always at least one feasible set of vehicle schedules which can be represented exactly in the y_{qr} without generating slack in the service-covering constraints. This solution is precisely the VSP2 solution itself.

3.5 Imposing equality in the service-covering constraints

The service-covering constraint rows could also be expressed as equalities, as follows:

$$\sum_{(qr)\in B_p} y_{qr} = s_p \quad , \quad p \in P \tag{3.16}$$

The absence of slack in these constraints is equivalent to a requirement that the level of service be independent of the manpower solution. The structure of the HM2 model is such that this change has no effect on the value of the objective function. To see this, first note that the cost coefficient associated with each of the y_{qr} variables in the HM2 model is zero. Thus during the course of the simplex algorithm any change of basis must always involve a change in the values of the $x_{ij,mn}$ variables if the value of the objective function is to decrease. In particular, if in passing from one

basis to another the simplex algorithm can reduce the objective function by introducing slack, such slack must always involve surplus activity in some constraint involving the manpower scheduling variables. On the other hand, it will never be necessary for there to be such surplus activity in any constraint involving exclusively the y_{qr} because the inequaltities (3.12) will always be sufficient to accomodate any surplus activity in the $x_{ij,mn}$. Thus in fact no model flexibility is lost by imposing the equality constraints (3.16).

The use of equality in the service-covering constraints (3.16) simplifies the HM2 model somewhat, in that under certain circumstances (3.16) is equivalent to a requirement that certain subsets of the y_{qr} sum to known quantities. Consider, as in Section 3.4, the set K of start periods in the VSP2 solution and the set L of end periods in that solution. We make the assumption, for now, that there is no period $q \in K$ such that $q = r + 1$ for some $r \in L$. Let R_q equal the number of vehicle schedules which begin in period q in the VSP2 solution, for all $q \in K$. Similarly, let C_r equal the number of vehicle schedules which end in period r in that solution. Clearly, the R_q and C_r are integer, as are the s_p of equations (3.13). Moreover, by our temporary assumption,

$$R_q = \begin{cases} s_q & , q \text{ the first period of the day} \\ s_q - s_{q-1} & , \text{otherwise} \end{cases} \qquad (3.17)$$

and

$$C_r = \begin{cases} s_r & , r \text{ the last period of the day} \\ s_r - s_{r+1} & , \text{otherwise} \end{cases} \qquad (3.18)$$

Applying (3.16) to (3.17) we have that, in all cases,

$$R_q = \sum_{(qr)\in B_{q'}} y_{qr} \qquad (3.19)$$

since $B_q = B_{q'}$ for q the first period of the day. In other words, the sum of all variables y_{qr} for q equal to a particular fixed period is always equal to a known integer, the number of blocks which begin at that period in the VSP2 solution.

Similarly, if we define the set

$$B_{r''} = \{ (kl) \in B_r : l = r \}$$

we can apply (3.16) to (3.17) to yield

$$C_r = \sum_{(qr)\in B_{r''}} y_{qr} \qquad (3.20)$$

since $B_r = B_{r''}$ for r the last period of the day. Thus the sum of all variables y_{qr} for r equal to a particular fixed period is always equal to the number of blocks which end at that period in the VSP2 solution.

To summarize, the equality constraints (3.16) imply (under our temporary assumption) that in the optimal solution HM2 the total number of potential vehicle schedules y_{qr} for some fixed q and this total for some fixed r are always equal to known constants.

If in fact there is some period $q \in K$ such that $q = r + 1$ for some $r \in L$, then the relations (3.17) and (3.18) no longer hold. Let q^* be a specific period in K such that $q^* = r^* + 1$ for some $r^* \in L$. Now (3.17) becomes

$$R_{q^*} = C_{r^*} + (s_{q^*} - s_{r^*})$$

and (3.18) becomes

$$C_{r^*} = R_{q^*} + (s_{r^*} - s_{q^*})$$

In other words, the number of vehicle schedules which must begin at q^* and the number which must end at r^* no longer follow simply from the relationships between the s_p .

We resolve this difficulty in practice by explicity adding a requirement to the HM2 model that

$$R_{q^*} = \sum_{(qr)\varepsilon B_{q^*}} y_{qr} \qquad (3.21)$$

for all q^* such that $q^* = r^* + 1$ for some $r^* \varepsilon L$. To do so introduces a measure of regularity into HM2 by requiring that the total number of vehicle schedules estimated by HM2 will always be equal to the total number which exist in the VSP2 solution.

The equality constraints (3.16), together with (3.21) where necessary, have the effect of reducing the ability of the y_{qr} variables to vary among themselves because certain subsets always sum to known quantities. In general, if n_B is the number of elements of B , n_K is the number of elements of K , and n_L is the number of elements of L , the total degrees of freedom among the y_{qr} will be equal to

$$D = n_B - (n_K + n_L - 1) .$$

This completes the discussion of the HM2 linear model. Chapter 4 uses the Apple Grove scheduling problem to give an extended example of this linear program. Chapter 5 presents a heuristic technique for using HM2 to adjust an initial vehicle solution to produce another which is more efficiently served with manpower.

Chapter 4

An example of the HM2 linear program

4.1 Introduction

This chapter presents an example of the HM2 linear program. We make use of the Apple Grove scheduling example introduced in Chapter 2 to fully illustrate the way in which the model is set up. Chapter 5 continues the exposition with a method to create vehicle schedules which are efficient in their utilization of manpower.

4.2 Obtaining a preliminary vehicle solution

The initial vehicle solution in this research is found using VSP2, a network flow model which minimises a composite objective function. The VSP2 model was introduced in Chapter 2, where the Apple Grove scheduling example was used to illustrate it. Recall (see Table 2.3) that VSP2 created three all-day vehicle blocks, three which operate only in the morning and three which operate only in the evening. Thus the total fleet size is six vehicles. Recall as well that the total platform time in that solution is 31 hours and 45 minutes.

4.3 Assigning discrete periods and forming a service histogram

Like the linear program HM1 used in the system HASTUS, the rows and columns of the new linear program HM2 are generated using a time scale broken up into discrete periods. The service-covering constraints (3.16), for example, give the minimum coverage requirements for individual periods during the service day. Since the vehicle schedules themselves are created using an

effectively continuous time scale, the information which they contain must be discretized to be used by the linear program.

For reasons of compatibility with existing methods, this research uses a discretization procedure which is essentially the same as that used in the HASTUS-Macro package (GIRO Inc., 1983). The procedure is as follows. Let H be an integer equal to a period length in minutes. This value can be chosen at will, although clearly the smaller the value of H the more accurate the representation of the information in the set of vehicle schedules. Now let V be the number of vehicle schedules in the VSP2 solution, and let B_v and E_v denote the start and end times, also in minutes, for vehicle schedule v, for $v = 1,\ldots,V$. Set S denote an integer that $0 \leq S \leq H$. We find the integer values

$$B_v^* = [(B_v - S)/H] + 1 \qquad (4.1)$$

$$E_v^* = [(E_v - S)/H] \qquad (4.2)$$

where the operator $[.]$ indicates a truncation of the fractional portion of the ratio. The discretization procedure amounts to nothing more than assuming that vehicle schedule v begins at period B_v^* and extends over $(E_v^* - B_v^* + 1)$ periods to end at period E_v^*.

The parameter S represents a shift in the time of origin of each of the periods. If $S = 0$ we begin period v at midnight plus $(v-1)H$ minutes. If $S = 30$ we begin each period at 12:30 a.m. plus $(v-1)H$ minutes.

Such discretization procedure will naturally distort the information in the vehicle schedule solution somewhat. It is possible to mitigate this distortion in practice, however, through the judicious choice of a period length. For example, any vehicle schedule can be more accurately represented

the shorter the period length. Importantly, however, the same periods must serve both to discretize the vehicle schedule and generate the manpower scheduling columns of HM2, that is the elements $(ij,mn) \in D$. GIRO Inc. (1983) comments on the effect that period length might have on the ease with which one can simulate the possible duty types permitted by a certain collective agreement.

Once vehicle blocks have been discretized a service histogram, which provides the constants s_p of (3.16), is available immediately. s_p, the height of the histogram at period p, is equal to the number of vehicle schedules v such that

$$B_v^* < p < E_v^*$$

In the Apple Grove scheduling example we choose a period length E equal to 60 minutes and shift S equal to 30 minutes. Thus period 1 is from 12:30 a.m. to 1:29 a.m., period 2 is from 1:30 to 2:29 a.m., and so on. We use eight periods in this example, from period 8 to period 15 inclusive, as follows. The earliest pull-out is at 7:30 a.m. (450 minutes after midnight). Letting $S = 30$ and $H = 60$ in (4.1) we have

$$[(450 - 30)/60] + 1 = 8$$

Similarly, the last pull-in of the day is at 3:30 p.m. (930 minutes after midnight). Making the appropriate substitutions in (4.2) yields

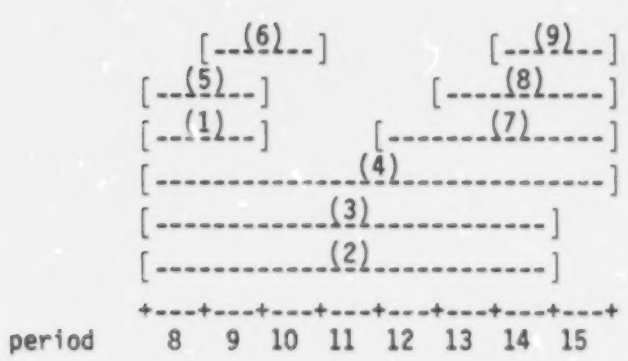
$$[(930 - 30)/60] = 15$$

The top half of Figure 4.1 represents the VSP2 blocks after the application of the discretization. There is in fact some distortion of the block solution, which shows up in the increase in presumed platform time.

Figure 4.1

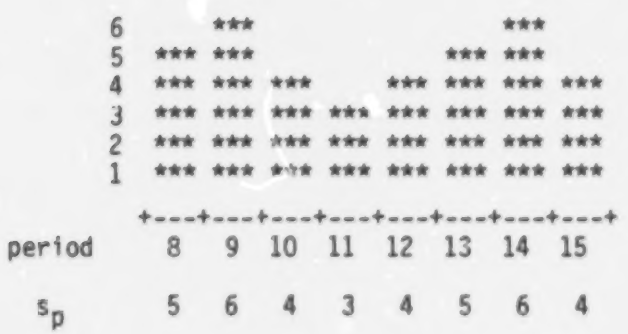
Results of discretization of VSP2 solution
in Apple Grove example

a. Discrete vehicle schedules



Note: Block numbers from Table 2.3 are in parentheses

b. Service histogram



While there is a total of 36 hours and 25 minutes in the VSP2 solution, there are 37 hours of assumed platform time in the discretization. The bottom half of Figure 4.1 is the histogram which arises from the discretized blocks. The s_p are reported at the bottom of the histogram. We note at this point that the set K is

$$K = \{ 8,9,12,13,14 \}$$

and that the set L is

$$L = \{ 9,10,14,15 \}$$

The set B is immediately available as the set of all (qr) pairs such that $q \in K$, $r \in L$, and $q \leq r$, that is,

$$B = \{ (8,9),(8,10),(8,14),(8,15),(9,9),(9,10),(9,14),(9,15),$$
$$(12,14),(12,15),(13,14),(13,15),(14,14),(14,15) \}$$

We also note that there is no $q \in K$ such that $q = r + 1$ for some $r \in L$. Thus we will need no constraints of the form (3.21) in our linear program.

4.4 Generating the HM2 coefficient matrix

As is the case with the HM1 linear program (Blais and Rousseau, 1982), the HM2 coefficient matrix is generated according to parameters representing the collective agreement governing labor practices at a transit property. In HM2, however, we have to take into account variable vehicle schedules when we set up the feasibility constraints for the linear program. In particular, there will in general be more small-block constraint rows in HM2 than in any particular realization of HM1, because any of a number of potential small

blocks may be part of an optimal solution to HM2.

To illustrate the matrix generation process we return to the Apple Grove scheduling example and give, in Table 4.1, a set of fictitious work rules governing crew scheduling in Centreville. The maximum spread of any workday is seven hours. However, any workday which involves a spread of more than five hours must pay time-and-a-half on time worked between five and six hours, and double time on time worked between six and seven hours. The maximum total platform time which any single operator can work in a day is five hours. The day guarantee is four hours and the piece guarantee is two hours. Overtime is paid at time-and-one-half after four hours of work. There are at most two pieces with an unpaid break between them per workday. If there is such an unpaid break it must not be less than one hour long. Finally, 50% of all workdays must not involve any unpaid break at all.

The final entry in Table 4.1 is intended to limit the size of the problem in such a way that unlikely potential workdays are eliminated. We require that any potential two-piece workday must cover at least one of the two service peaks, which we can see from Figure 4.1 are at period 9 and at period 14. This eliminates possibilities where the unpaid break occurs during one of the peaks, a situation which would almost never be found in an optimal solution (Blais and Rousseau, 1982).

We are now in a position to build the HM2 coefficient matrix. We first consider its rows. For each of the "block-covering" constraints (3.6) we add a row corresponding to each period in the service day. The discretization procedure described in section 4.3 used eight hour-long periods for the Apple Grove example; thus there are eight "block-covering" constraint rows. These are the first eight rows in Table 4.2, those labelled BLOCK8, BLOCK9, ... BLOCK15 along the left-hand margin.

Table 4.1

Crew scheduling rules for the
Apple Grove example

Maximum spread	7:00
Spread premium	:30 after 5:00 of spread 1:00 after 6:00 of spread
Maximum platform time	5:00
Daily time guarantee	4:00
Piece guarantee	2:00
Overtime	:30 after 4:00 of platform time
Maximum pieces per duty	2
Minimum unpaid break in 2-piece duty	1:00
Maximum unpaid break in 2-piece duty	3:00
Maximum piece length in 2-piece duty	3:00
Percent of duties which must be one-piece	50%
Peak hours	9:00, 4:00

Any two-piece duty must cover at least one of the peaks.

Table 4.2

HM2 problem matrix for the Apple Grove example

	STR0810	STR0811	STR0812	STR0912	STR0913	STR1013	STR1014	STR1114	STR1115	STR1214	STR1215	STR1315	T0809112	T0809113	T0809123	T0809124	T0809134	T0810123	T0810134	T0810144	T0909112	T0909113
BLOCK8	1	1	1										1	1	1	1	1	1	1	1		
BLOCK9	1	1	1	1	1								1	1	1	1	1	1	1	1	1	1
BLOCK10	1	1	1	1	1	1	1											1	1	1		
BLOCK11		1	1	1	1	1	1	1	1				1	1							1	1
BLOCK12			1	1	1	1	1	1	1	1	1		1	1	1	1		1			1	1
BLOCK13					1	1	1	1	1	1	1	1		1	1	1	1	1	1			1
BLOCK14							1	1	1	1	1	1				1	1		1	1		
BLOCK15									1		1	1										
STRAIGHTS	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
SB0809												1	1	1	1	1	1					
SB0810	1																	1	1	1		
SB0909																					1	1
SB0910																						
SB1214										1						1						
SB1314																	1		1			
SB1315												1										
SB1414																				1		
SB1415																						
SERVICE8																						
SERVICE9																						
SERVICE10																						
SERVICE11																						
SERVICE12																						
SERVICE13																						
SERVICE14																						
SERVICE15																						
COSTROW	4.0	4.0	5.5	4.0	5.5	4.0	5.5	4.0	5.5	4.0	4.0	4.0	4.0	6.0	4.5	7.0	5.5	6.0	7.0	6.5	4.0	5.0

Table 4.2 (continued)

	T 0 9 0 9 1 2 1 3	T 0 9 0 9 1 2 1 4	T 0 9 0 9 1 3 1 4	T 0 9 0 9 1 3 1 5	T 0 9 0 9 1 4 1 4	T 0 9 0 9 1 4 1 5	T 0 9 1 0 1 2 1 3	T 0 9 1 0 1 2 1 4	T 0 9 1 0 1 3 1 4	T 0 9 1 0 1 3 1 5	T 0 9 1 0 1 4 1 4	T 0 9 1 0 1 4 1 5	T 0 9 1 1 1 3 1 4	T 0 9 1 1 1 4 1 4	T 0 9 1 1 1 4 1 5	T 1 0 1 1 1 3 1 4	T 1 0 1 1 1 3 1 5	T 1 0 1 1 1 4 1 4	T 1 0 1 1 1 4 1 5	T 1 0 1 1 2 1 4 4	T 1 0 1 1 2 1 4 5	T 1 1 1 1 2 1 4 4	T 1 1 1 1 2 1 4 1 5
BLOCK8																							
BLOCK9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1								
BLOCK10							1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
BLOCK11													1	1	1	1	1	1	1	1	1	1	1
BLOCK12	1	1					1	1												1	1	1	1
BLOCK13	1	1	1	1			1	1	1	1			1			1	1						
BLOCK14		1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
BLOCK15				1		1				1		1			1		1		1		1		1
STRAIGHTS	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
SB0809																							
SB0810																							
SB0909	1	1	1	1	1	1																	
SB0910							1	1															
SB1214								1															
SB1314			1						1				1			1							
SB1315				1						1							1						
SB1414					1						1			1				1		1		1	
SB1415						1						1			1				1		1		1
SERVICE8																							
SERVICE9																							
SERVICE10																							
SERVICE11																							
SERVICE12																							
SERVICE13																							
SERVICE14																							
SERVICE15																							
COSTROW	4.0	5.5	4.5	6.5	4.5	5.5	4.0	6.0	4.5	7.0	4.5	5.5	6.0	5.5	7.0	4.0	6.0	4.0	4.5	5.0	6.0	4.0	4.0

Table 4.2 (conclusion)

	VEH0809	VEH0810	VEH0814	VEH0815	VEH0909	VEH0910	VEH0914	VEH0915	VEH1214	VEH1215	VEH1314	VEH1315	VEH1414	VEH1415		RHS
BLOCK8	-1	-1	-1	-1											≥	0
BLOCK9	-1	-1	-1	-1	-1	-1	-1	-1							≥	0
BLOCK10		-1	-1	-1		-1	-1	-1							≥	0
BLOCK11			-1	-1			-1	-1							≥	0
BLOCK12			-1	-1			-1	-1	-1	-1					≥	0
BLOCK13			-1	-1			-1	-1	-1	-1	-1	-1			≥	0
BLOCK14			-1	-1			-1	-1	-1	-1	-1	-1	-1	-1	≥	0
BLOCK15				-1				-1		-1		-1		-1	≥	0
STRAIGHTS															≥	0
SB0809	-1														≥	0
SB0810		-1													≥	0
SB0909					-1										≥	0
SB0910						-1									≥	0
SB1214									-1						≥	0
SB1314											-1				≥	0
SB1315												-1			≥	0
SB1414													-1		≥	0
SB1415														-1	≥	0
SERVICE8	1	1	1	1											≥	5
SERVICE9	1	1	1	1	1	1	1	1							≥	6
SERVICE10		1	1	1		1	1	1							≥	4
SERVICE11			1	1			1	1							≥	3
SERVICE12			1	1			1	1	1	1					≥	4
SERVICE13			1	1			1	1	1	1	1	1			≥	5
SERVICE14			1	1			1	1	1	1	1	1	1	1	≥	6
SERVICE15				1				1		1		1		1	≥	4
COSTRON																

To construct the "small-block" constraints (3.7) we have to know the composition of the set S. This requires in turn that we know the value of the constant W, introduced in Chapter 3, which defines a "small block". Recall that such a block is one which should not be cut into smaller pieces of work as part of a manpower solution. The reason for such a restriction, however, arises from the piece guarantee, or minimum amount of time which must be paid to an operator between any two unpaid periods. In the Apple Grove problem this piece guarantee is two hours. As there is no way to cut any block of three hours or less without creating at least one piece which requires payment of a piece guarantee, we define a "small block" as any block which is less than or equal to three hours (periods) in length.

The set B arises directly from an analysis of the VSP2 solution and was enumerated in section 4.3. The set S is simply that subset of B in which all pairs (qr) are such that $(r - q + 1) < 3$, that is, all pairs corresponding to blocks of up to three hours' (periods') length. This set is as follows:

$$S = \{(8,9),(8,10),(9,9),(9,10),(12,14),$$
$$(13,14),(13,15),(14,14),(14,15)\}$$

We add eight rows to the HM2 matrix in table 4.2, one for each member of S. These are labelled SB0809, SB0810, ... , SB1415.

There are eight rows in Table 4.2 corresponding to the "service-covering" constraints (3.16), one for each period. These are labelled SERVICE8, SERVICE9, ... , SERVICE15. In addition, there is a row corresponding to the constraint on the percentage of workdays (50%) which contain no unpaid break. This single constraint is the only "supplemental" constraint (3.5) in the Apple Grove problem and is labelled STRAIGHTS. Finally, there is the row

containing the coefficients of the objective function, which is labelled COSTROW.

We have noted in section 4.3 that there is no $q \in K$ such that $q = r + 1$ for some $r \in L$. Thus, from the discussion of section 3.5, we conclude that as a practical matter we can exclude pull-out constraints of the form (3.4) in the HM2 problem matrix, in addition to constraints of the form (3.21).

Now consider the columns of the HM2 coefficient matrix. These columns are generated in accordance with the set of work rules in Table 4.1, following a column generation algorithm due to Blais and Rousseau (1982) and which is presented in Table 4.3(a). The naming convention for these columns is as follows. Consider a column representing $(ij,mn) \in D$. If $m = n = 0$, the name of the column is STRij to indicate that is represents a straight duty, or a duty with no unpaid break. If $m \neq 0$, the name of the column is Tijmn to indicate a two-piece duty, or a duty which includes an unpaid break. A column representing $(qr) \in B$ has the name VEHqr. In all cases leading zeroes are kept on the indices. Thus the column representing (ij,mn) for $i = 8$, $j = 9$, $m = 11$, and $n = 12$ is T08091112.

The first twelve columns of Table 4.2 are those which represent the potential straight duties in the Apple Grove problem. Note that the columns labelled STR0810 through STR1215 in fact represent all possible straight duty prototypes of between four and five hours, but that there are only three such prototypes of three hours (STR0810, STR1214, and STR1315) and none of less than three or more than five hours. The reason for this can be found in the work rules of Table 4.1. According to these rules, the minimum pay for any regular workday is four hours, and the maximum allowed platform time is five hours. Thus we do not include columns with nonzero coefficients in more than

Table 4.3

Column generation procedures for the
HM2 problem matrix in the Apple Grove example

a. Procedure for column representing $(ij,mn) \in D$

1. For $i < p < j$ place a 1 in the p^{th} block-covering constraint row. If $m = 0$ go to step 2. If not, for $m < p < n$ place a 1 in the p^{th} block-covering constraint row.
2. If $(i,j) \in S$ place a 1 in the $(ij)^{th}$ small-block constraint row. If $m = 0$ go to step 3. If not, and if $(mn) \in S$, place a 1 in the $(mn)^{th}$ small-block constraint row.
3. If $m = 0$ place a 1 in the row representing the constraint on straight duties. If not, place a -1 in that row.

b. Procedure for column representing $(qr) \in B$

1. For $q < p < r$ place a -1 in the p^{th} block-covering constraint row and a 1 in the p^{th} service-covering constraint row.
2. If $(qr) \in S$ place a -1 in the $(qr)^{th}$ small-block constraint row.

five consecutive block-covering constraint rows because in Apple Grove we cannot use duties which run longer than five hours. We do not include most columns representing duties of less than four hours because such duties would be costed at the same rate as a workday of four hours. If in fact such a short workday were part of an optimal solution, the HM2 linear program could represent this easily by introducing slack in one or more of the block-covering constraints. The three exceptions arise because each corresponds to a potential small block as well as a potential workday, and the only way to adequately represent the small-block constraints is to include these three columns.

Finally, each objective function coefficient in the row labelled COSTROW gives the number of hours paid, according to Table 4.1, for a prototype workday of the length implied by the configuration of coefficients in its column. Each of the first 12 columns of Table 4.2 are costed at 4.0 hours except those of five hours in length, which include an overtime premium of one-half hour applicable to the fifth hour of the workday.

The columns representing split duties (T08091112 through T11121415) are somewhat more complicated than those representing the straight duties. We now have to respect a three-hour maximum on piece length, a two-hour piece guarantee, and a seven-hour maximum spread. For the same reason that we can ignore most columns representing straight duties of less than three hours, we can forego most columns which represent duties on which at least one piece of work lasts less than two hours. The only exceptions to this rule are those pieces which correspond exactly to potential small blocks, as the only way that we can ensure that the small-block constraints (3.7) are formulated adequately is by including columns containing these pieces.

The columns representing split duties are also assigned objective

function coefficients in accordance with the remuneration rules in Table 4.1. A few examples will clarify how these rules are used in costing out these columns. T08101414 comprises a three-hour first piece and a one-hour second piece (which corresponds to a potential block). Thus the basic pay for the duty which this column represents is five hours, since we have to pay a minimum of two hours for any piece. The spread, however, is seven hours. Thus we have to pay a premium of one-half hour for the sixth hour (even if this hour is not even worked) and one hour for the seventh hour. Thus the total cost comes to 6.5 hours. By contrast, column T08101213 comprises a three-hour first piece and a two-hour second piece. The basic pay for this duty is five hours. However, a half-hour premium is paid to account for the fact that there are five hours worked instead of four (an overtime premium) and another half-hour premium is added because the total spread is six hours. Thus the total cost for this duty comes to 6.0 hours. Finally, the column T09101315 is paid at a basic rate of five hours plus the half-hour overtime premium, a half-hour premium for the sixth hour of spread and a full-hour premium for the seventh hour of spread, for a total cost of 7.0 hours.

The final 14 columns of the HM2 problem matrix represent the elements $(qr) \in B$ and are generated according to the rules in Table 4.3(b).

The constraint on the total number of straight duties (the row labelled STRAIGHTS) arises from the condition in Table 4.1 is that at least 50% of all duties must be straight duties. By placing, in the STRAIGHTS row, a 1 in columns corresponding to duties with no unpaid break and a -1 in columns corresponding to duties with such a break, and then requiring that the total row activity be nonnegative, we ensure that there will be at least as many straight duties as split duties represented in the optimal solution.

The vector of constraint constants of the problem matrix in Table 4.2 is

established directly from inequalities (3.12) and (3.13) and equalities (3.16) of Chapter 3. The only nonzero elements are those in the service-covering constraint rows. These elements are the s_p, which we have found from discretizing the VSP2 vehicle block solution in section 4.3.

We can see immediately the effect that the equation set (3.16) has on the ability of the y_{qr} to vary in the Apple Grove example. Consider Figure 4.2, which is a two-dimensional tableau showing the relationships between the y_{qr} variables in the Apple Grove problem. The five elements in K form the left margin of this tableau, labelled "start periods". Similarly, the four elements in L form the top margin of the tableau, labelled "end periods". There is a y_{qr} variable corresponding to "start period" q for $q \in K$ and "end period" r where $r \in L$, for all cases where $q < r$. In the Apple Grove problem, there are exactly 14 such variables. Along the right-hand margin of the tableau are the R_q, the totals to which the y_{qr} in any row must sum. Along the bottom margin are the C_r, the totals to thich the y_{qr} variables in any column must sum. Because of these constraints there are only six degrees of freedom in the tableau, or only six y_{qr} which can be fixed independently before the rest of the tableau follows from the equality constraints (3.16).

Figure 4.3 presents the discretized VSP2 solution using the two-dimensional tableau format of Figure 4.2. From Figure 4.3 it is easy to see the origin of the row and column totals used in Figure 4.2.

The two-dimensional tableau format is a convenient way to represent an ensemble of y_{qr} variable values and we will resort to it frequently during the course of this exposition.

Figure 4.2

Admissible vehicle variables
in Apple Grove example

Start periods \ End periods	9	10	14	15	Row totals R_q
8	$y_{8,9}$	$y_{8,10}$	$y_{8,14}$	$y_{8,15}$	5
9	$y_{9,9}$	$y_{9,10}$	$y_{9,14}$	$y_{9,15}$	1
12			$y_{12,14}$	$y_{12,15}$	1
13			$y_{13,14}$	$y_{13,15}$	1
14			$y_{14,14}$	$y_{14,15}$	1
Column totals C_q	2	1	2	4	9

Total vehicle variables = 14

Total degrees of freedom = 6

Figure 4.3

Discrete VSP2 solution in Apple Grove example presented in a 2-dimensional tableau

Start period \ End period	9	10	14	15	Row totals
8	2	0	2	1	5
9	0	1	0	0	1
12			0	1	1
13			0	1	1
14			0	1	1
Column totals	2	1	2	4	9

4.5 Solving the HM2 linear program

The HM2 linear program is easily solved using existing linear program solution packages, in the case of this research the Control Data APEX-III package (Control Data Corporation, 1979). To facilitate comparison with other solutions, we also find a solution to HM2 which is integer in the y_{qr}. We obtain such a solution using the Mixed-Integer Programming option of APEX-III. This option uses a branch-and-bound algorithm (Tomlin, 1970) to find the optimal solution to a linear program which is integer in a certain subset of the problem variables. In our case, we place integrality constraints on the y_{qr} variables and solve the mixed-integer programming problem. The results are reported in Figure 4.4. Note that we have found an optimal solution to HM2 which is integer in the y_{qr}. Also note that the number of integrality constraints is much smaller than the actual number of y_{qr} variables. This is because (3.16) has reduced the total number of degrees of freedom among the y_{qr} variables, including the freedom not to be integer. Thus we need only six integrality constraint to guarantee an integer solution.

If we fix the y_{qr} at a predetermined set of values, we can use the HM2 model to estimate the cost of serving a vehicle schedule solution consistent with that set of y_{qr} values. This concept is in fact not new, for HM2 with the y_{qr} fixed at constant values is nothing more than the HM1 model, which is in current use (see (3.9) to (3.11) in Chapter 3). With this device, however, we can evaluate the relative efficiency of the VSP2 solution with repect to the HM2 optimum.

In the summary at the bottom of Figure 4.4, we note that the value of the HM2 objective function with the y_{qr} fixed at the discretized VSP2 solution is 2.4% above that which occurs at either the continuous or mixed-integer HM2 solution with the y_{qr} free to vary. Thus we see that the linear manpower

Figure 4.4

HM2 mixed-integer solution (vehicle variables)
in Apple Grove example

Start period \ End period	9	10	14	15
8	2.0	0.0	1.0	2.0
9	1.0	1.0	0.0	0.0
12			0.0	1.0
13			1.0	0.0
14			0.0	1.0

Summary statistics

Number of rows	31	
Number of columns	62	
Matrix density	27.1%	
No. of y_{qr} variables	14	
No. of integrality const.	6	
Solution times (CDC Cyber/855)		
to continuous solution	.77	sec.
to MIP solution	.27	sec.
Objective at optimum	41.5	
Objective with y_{qr} variables fixed at VSP2	42.5	
Difference	1.0	(2.4%)

cost model HM2 suggests that there might exist vehicle schedules which are more efficient than those created using VSP2.

It is worthwhile to briefly examine why fixing the y_{qr} at those of Figure 4.3 increases the value of the objective function in HM2. Since only the $x_{ij,mn}$ carry any cost in the HM2 problem we have to examine those $x_{ij,mn}$ variables which take nonzero values in any solution to HM2. These $x_{ij,mn}$ variables give the number and type of operator workdays that HM2 would use to cover a vehicle schedule solution -- either given a priori by fixing the y_{qr} at predetermined values of found by HM2 itself using variable y_{qr} Table 4.4 presents these variables with the value and unit cost of each for the obtained optimal solution to HM2 and for that solution with the y_{qr} fixed at the VSP2 solution and Table 4.5 summarizes these solutions by grouping the $x_{ij,mn}$ variables together according to the platform and spread which each represents. HM2 used ten duties to cover the fixed VSP2 solution while it only used 9.5 when the vehicle solution was allowed to vary. Also, although there are roughly six straight duties in both solutions, half of these involve only three hours of platform time in the solution with y_{qr} fixed at VSP2 while the variable-block solution uses more straight duties of the full 4-hour length. In addition, the variable-block solution uses two-piece duties of longer spread than in the fixed-block solution, even though those duties carry higher costs. In fact, the average cost per duty in the variable-block solution is 4.34 hours, while that in the fixed-block solution is 4.2 hours. In other words, the effect of introducing variable vehicle schedules into the Apple Grove scheduling example seems to be to take advantage of longer spreads to reduce the required number of operators, even if to do so means a higher cost paid per operator.

Before leaving this example of the HM2 linear program, we present in

Table 4.4

Manpower variables at nonzero values
Apple Grove example

a. Mixed-integer HM2 solution

Variable	Value	Cost
STR0810	1.0	4.0
STR0811	2.0	4.0
STR1215	3.0	4.0
T08091112	.5	4.0
T08091314	1.5	5.5
T09101314	.5	4.5
T09101415	.5	5.5
T11121415	.5	4.0

b. Vehicle variables fixed at VSP2

Variable	Value	Cost
STR0810	2.0	4.0
STR0811	1.0	4.0
STR1215	2.0	4.0
STR1315	1.0	4.0
T08091112	1.0	4.0
T08091314	1.0	5.5
T09101314	1.0	4.5
T11121415	1.0	4.0

Table 4.5

Summary of manpower variables
Apple Grove example

a. Mixed-integer HM2 optimum

	Value	Cost	Value x cost
Straight duties			
Platform 3 hours	1.0	4.0	4.0
Platform 4 hours	5.0	4.0	20.0
Two-piece duties			
Platform 4 hours			
Spread 5 hours	1.0	4.0	4.0
Spread 6 hours	.5	4.5	2.25
Spread 7 hours	2.0	5.5	11.0
Total	9.5		41.25

b. Vehicle variables fixed at VSP2

	Value	Cost	Value x cost
Straight duties			
Platform 3 hours	3.0	4.0	12.0
Platform 4 hours	3.0	4.0	12.0
Two-piece duties			
Platform 4 hours			
Spread 5 hours	2.0	4.0	8.0
Spread 6 hours	1.0	4.5	4.5
Spread 7 hours	1.0	5.5	5.5
Total	10.0		42.0

Table 4.6 a summary comparison of the VSP2 and mixed-integer HM2 solutions. This table is actually a rearrangement of the material in Figures 4.3 and 4.5. The column marked "end period" is a list of all the columns in Figures 4.3 and 4.5. The other two columns are simply disaggregations of the contents of the cells of those two figures. What is remarkable in this comparison is that there is only a single difference between the two solutions: the VSP2 solution includes a vehicle schedule from period 8 to period 14, one from 8 to 15, and one from 13 to 15, while the mixed-integer HM2 solution calls for a vehicle schedule from period 13 to period 14 and two from 8 to 15.

Chapter 5 is concerned with the problem of making changes to a vehicle schedule solution in order to produce another which "costs out" more advantageously in terms of the HM2 model. We will see, in fact, that a single minor change to the VSP2 solution will produce a set of vehicle schedules which HM2 regards as optimal.

Table 4.6

A summary comparison of the VSP2 and HM2 mixed-integer solutions, Apple Grove example

End Period	VSP2 starts	HM2/MI starts	Sole Difference
9	8	8	
	8	8	
10	9	9	
14	8	8	
	8	13	←
15	8	8	
	12	8	←
	13	12	
	14	14	

Chapter 5

Creating vehicle schedules which use manpower efficiently

5.1 Introduction

This chapter presents and illustrates the problem of using HM2 to find efficient vehicle schedules and proposes a heuristic for creating such schedules. The general strategy introduced in this chapter is one of marginally improving the initial VSP2 solution using information available from the HM2 linear model. We begin the chapter with a discussion of the basic concepts of the marginal improvement problem. We then illustrate these concepts using the Apple Grove example and propose a heuristic method for applying them generally to a set of data.

5.2 A conceptual framework for creating efficient vehicle schedules

This section presents some background to the problem of creating efficient vehicle schedules. First we state precisely what is meant by a rehooking of a vehicle schedule solution. To do this we introduce two new terms related to vehicle schedules. For any trip $i \in N$, the successor to i, denoted by SU(i), is that trip which immediately follows trip i in a vehicle schedule. If i is the last trip in a block, we define SU(i) to be equal to zero. Similarly, the predecessor to i, denoted PR(i), is the index of that trip which immediately precedes trip i in a vehicle schedule. Again, if i is the first trip in a block we define PR(i) to be equal to zero. In Chapter 2 we defined a hook between trips i and j to mean that trip j follows trip i in a vehicle schedule. In terms of the new nota-

tion, we say that trips i and j are hooked if $SU(i) = j$ and $PR(j) = i$.

Note that $PR(i)$ and $SU(i)$ are inverse functions, but that neither is defined for $i = 0$. Thus if $SU(i) \neq 0$ then $i = PR(SU(i))$ and if $PR(i) \neq 0$ then $i = SU(PR(i))$.

A rehooking is nothing more than a switch of successors and predecessors between a series of trips. For example, consider three trips i, j, and k which form part of a vehicle solution, and assume that trips i and k are hooked but that trip j is the first trip in a block. Then $SU(i) = k$, $PR(k) = i$, and $PR(j) = 0$. Figure 5.1(a) illustrates this situation.

A rehooking of these trips places j after i in a vehicle schedule and makes k the first trip in another. Thus $PR(k) = 0$, $SU(i) = j$, and $PR(j) = i$, as illustrated in Figure 5.1(b).

Note that we have not changed any other hooks in this particular vehicle schedule solution. Thus $PR(i)$, $SU(j)$, and $SU(k)$ would remain unchanged, as would all other successors and predecessors.

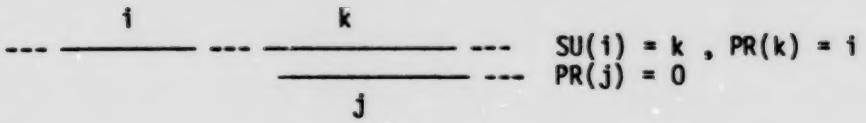
Next we note that in general a rehooking will change the pattern in which vehicle schedules begin and end in a vehicle schedule solution. For example, say that before the rehooking trips i and k form part of a vehicle schedule which begins at 8:03 and ends at 14:29, and that trip j begins one at 10:01, as in Figure 5.1(c). After the rehooking trip k begins a vehicle schedule at 9:59 which ends at 14:29. Also, trips i and j form part of a vehicle schedule which begins at 8:03 and ends at 12:46. The new situation is illustrated in Figure 5.1(d).

From the development of Chapter 3, however, we know that the pattern in which vehicle schedules begin and end determines the constraint constants in the HM1 linear model (see Section 3.3). Thus if we were to "cost out" the vehicle solutions in Figure 5.1(c) and (d) with the HM1 linear model, we would

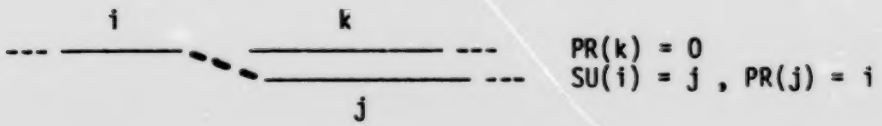
Figure 5.1

Example of a trip rehooking
in a vehicle solution

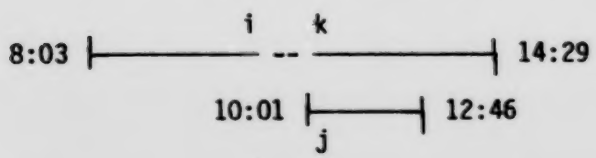
(a) Trips before rehooking



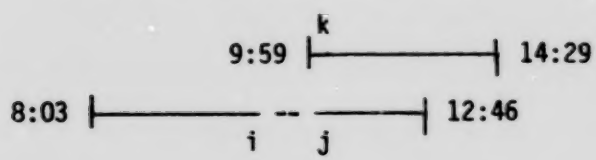
(b) Trips after rehooking



(c) Blocks before rehooking



(d) Blocks after rehooking



not expect the two cost estimates to be the same.

The HM2 model substitutes linear combinations of y_{qr} variables for the constraint constants of HM1, thus in effect making these constraint constants variable as well. But much as the HM1 model derives its constraint constants from a vehicle solution, we can "represent" or "model" the same vehicle schedule solution using a set of values of the y_{qr} such that the constraint constants are recreated as sums of these y_{qr}, in the sense of (3.9) to (3.11). Moreover, if we fix the y_{qr} at these values in the HM2 model we have effectively simulated the HM1 model, as we discussed in Section 4.5. Hence as the pattern of start and end times in a vehicle solution changes the values of the y_{qr} which model this solution change as well, and <u>a fortiori</u> the objective function of the HM2 model given such values of the y_{qr}.

We have already used, in Section 4.5, the device of fixing the y_{qr} at predetermined values to evaluate the efficiency of the Apple Grove VSP2 solution relative to the HM2 optimum.

Thus, by fixing the y_{qr} at predetermined values we can evaluate a vehicle solution using the HM2 model and hence assign an objective function value to it. Since a rehooking changes a vehicle solution and hence the y_{qr} values which model it, we can model the rehooking, and evaluate its cost impact, through the mechanism of passing from one basis to another in the HM2 model.

Before proceeding, we add a word of caution. We have to be sure that we can model a rehooked vehicle schedule using existing y_{qr}. If the set B consists of <u>all</u> (qr) pairs such that $q < r$ then this precaution does not aries, since then we can always find a (qr) pair which matches any discrete start and end times found using the procedure of Section 4.3. Once we

not expect the two cost estimates to be the same.

The HM2 model substitutes linear combinations of y_{qr} variables for the constraint constants of HM1, thus in effect making these constraint constants variable as well. But much as the HM1 model derives its constraint constants from a vehicle solution, we can "represent" or "model" the same vehicle schedule solution using a set of values of the y_{qr} such that the constraint constants are recreated as sums of these y_{qr}, in the sense of (3.9) to (3.11). Moreover, if we fix the y_{qr} at these values in the HM2 model we have effectively simulated the HM1 model, as we discussed in Section 4.5. Hence as the pattern of start and end times in a vehicle solution changes the values of the y_{qr} which model this solution change as well, and a fortiori the objective function of the HM2 model given such values of the y_{qr}.

We have already used, in Section 4.5, the device of fixing the y_{qr} at predetermined values to evaluate the efficiency of the Apple Grove VSP2 solution relative to the HM2 optimum.

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Before proceeding, we add a word of caution. We have to be sure that we can model a rehooked vehicle schedule using existing y_{qr}. If the set **B** consists of all (qr) pairs such that $q < r$ then this precaution does not arise, since then we can always find a (qr) pair which matches any discrete start and end times found using the procedure of Section 4.3. Once we

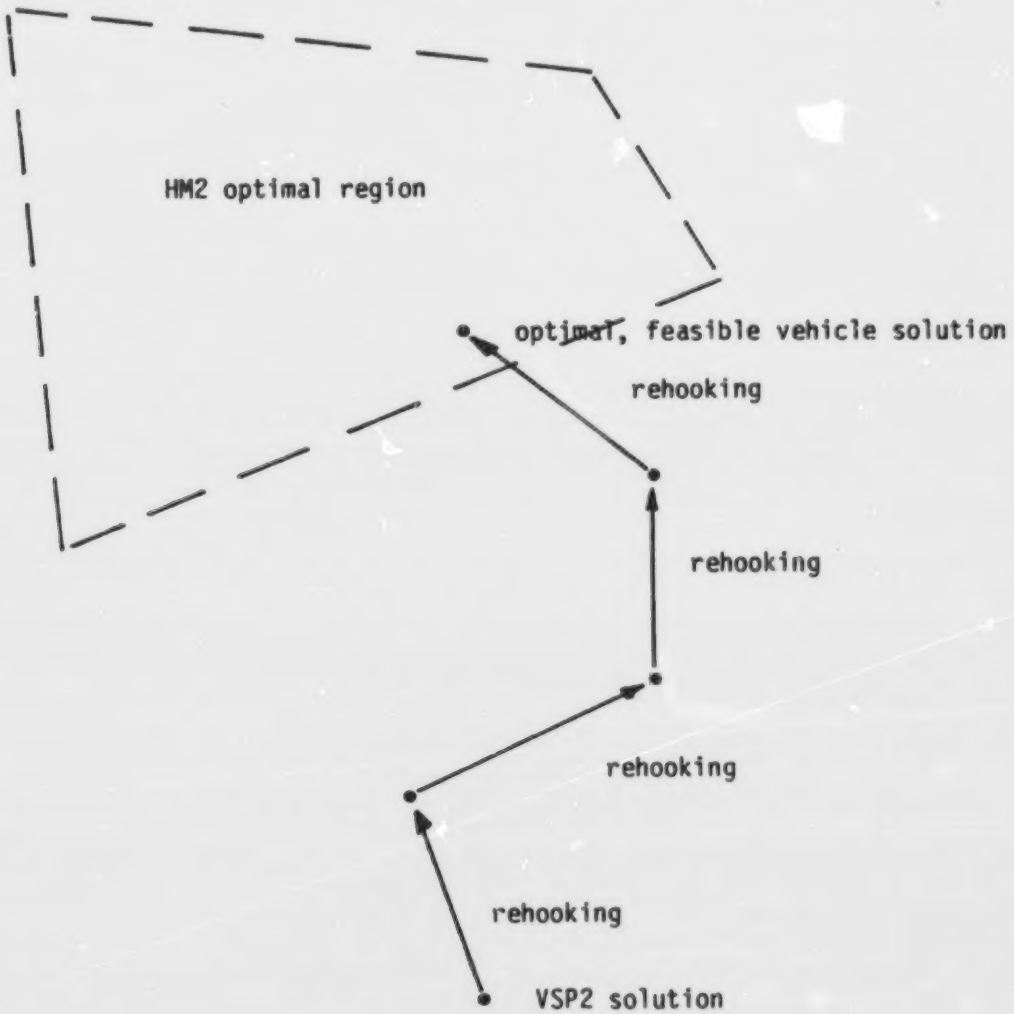
restrict the set **B** using the procedure of Section 3.4, however, we may find that a rehooked set of vehicle schedules yields, when discretized, a start period not in **K** or an end period not in **L**. For example, the vehicle schedules in Figure 5.1(c) begin at 8:03 and 10:01, respectively. If we were to discretize this solution using $H = 60$ and $S = 0$ in (4.1) we would find that the first block starts in period 9 and the second in period 11. If we were to do the same with the vehicle schedules in Figure 5.1(d), which begin at 8:03 and 9:59, respectively, we would find that the two starting periods were now 9 and 10. Thus the rehooking has caused us to "jump" a period boundary in a discrete vehicle schedule, even if there was only two minutes of platform time added to the solution.

As a practical matter, simple truncation of discrete vehicle blocks which "jump" a period boundary will suffice for reasonable cost estimation. We can, however, take account of the actual minutes added to a vehicle solution by a rehooking in evaluating its usefulness.

With this background we consider the problem of marginally improving feasible vehicle schedules, of which Figure 5.2 is a conceptual drawing. First, we note that there may be more than one optimal solution to the HM2 linear program. Second, we use the preceding discussion to represent feasible sets of vehicle schedules using feasible bases to HM2 and thus locate them in HM2 variable space with respect to the optimal region. The problem of marginal improvement is thus one of searching for alternate feasible vehicle solutions (using rehookings) which would translate into alternate HM2 bases, which in turn effectuate a decrease in the estimated cost of manpower. The large dots in Figure 5.2 indicate these alternate bases and the arrows between them downward movement in the HM2 objective function. A sequence of rehookings would ideally result in a feasible solution which is evaluated

Figure 5.2

Conceptual representation of the problem of creating efficient vehicle schedules



Note: The direction of the arrows represents a decrease in the values of the HM2 objective function

optimally using HM2, although this will not necessarily always be the case.

The next section will illustrate the phenomenon of rehooking a vehicle schedule solution to produce another which is evaluated at a lower cost. In Section 5.4 we present a heuristic for carrying out the search for rehookings in a larger set of data. This heuristic will be illustrated in Chapter 6 using trip timetable data from the public transport agency in Montréal.

5.3 An example of rehooking trips to improve a vehicle solution

We use the Apple Grove scheduling example to illustrate the vehicle schedule improvment problem. Table 5.1 is a detailed representation of the VSP2 solution for Apple Grove. This table sorts the trips in Table 2.1 according to end time (regardless of end location). The first column is a trip index i for each trip. The second column is PR(i), the index of the predecessor of trip i in the solution. The third and fourth columns contain the start and end times and locations of each trip. The fifth column is SU(i), the index of the successor of each trip i.

We noted in connection with Table 4.6 that there is only one pair of vehicle schedules in the VSP2 solution which is inconsistent with the optimal mixed-integer HM2 solution in Apple Grove. It turns out, in fact, that we can rehook of only two trip hooks in the VSP2 solution and create another solution which is fully consistent with the mixed-integer HM2 results. In Table 5.1 solution trip 38, which arrives at Centreville at 13:10, is hooked to trip 43, which departs immediately for Apple Grove. Trip 44, however, departs Centreville ten minutes later, at 13:20. Trips 38 and 43 form part of a vehicle schedule which begins at period 8 (trip 3, which leaves Centreville at 8:00, hence from the garage at 7:50) and ends at period 14 (trip 59, which arrives at Centreville at 15:10 and at the garage at 15:20). Trip 44 forms

Table 5.1

An illustration of a trip rehooking
in the Apple Grove example

I	Pr(I)	Start Loc/ Time	End Loc/ Time	Su(I)	I	Pr(I)	Start Loc/ Time	End Loc/ Time	Su(I)
1	0	AG/ 7:45	CV/ 8:15	5	34	31	AG/12:00	CV/12:30	39
2	0	AG/ 8:00	CV/ 8:30	8	35	32	CV/12:10	AG/12:40	38
3	0	CV/ 8:00	AG/ 8:30	7	36	33	AG/12:20	CV/12:50	41
4	0	AG/ 8:10	CV/ 8:40	10	37	0	CV/12:25	AG/12:55	40
5	1	CV/ 8:15	AG/ 8:45	11	38	35	AG/12:40	CV/13:10	43(44)
6	0	AG/ 8:20	CV/ 8:50	12	39	34	CV/12:40	AG/13:10	42
7	3	AG/ 8:30	CV/ 9:00	14	40	37	AG/12:55	CV/13:25	46
8	2	CV/ 8:30	AG/ 9:00	13	41	36	CV/12:55	AG/13:25	45
9	0	AG/ 8:40	CV/ 9:10	16	42	39	AG/13:10	CV/13:40	48
10	4	CV/ 8:40	AG/ 9:10	15	43	38(0)	CV/13:10	AG/13:40	47
11	5	AG/ 8:50	CV/ 9:20	0	44	0(38)	CV/13:20	AG/13:50	49
12	6	CV/ 9:50	AG/ 9:20	0	45	41	AG/13:25	CV/13:55	52
13	8	AG/ 9:00	CV/ 9:30	18	46	40	CV/13:30	AG/14:00	51
14	7	CV/ 9:00	AG/ 9:30	17	47	43	AG/13:40	CV/14:10	54
15	10	AG/ 9:15	CV/ 9:45	20	48	42	CV/13:40	AG/14:10	53
16	9	CV/ 9:15	AG/ 9:45	19	49	44	AG/13:50	CV/14:20	56
17	14	AG/ 9:30	CV/10:00	22	50	0	CV/13:50	AG/14:20	55
18	13	CV/ 9:30	AG/10:00	21	51	46	AG/14:00	CV/14:30	58
19	16	AG/ 9:45	CV/10:15	23	52	45	CV/14:00	AG/14:30	57
20	15	CV/ 9:45	AG/10:15	24	53	48	AG/14:10	CV/14:40	60
21	18	AG/10:00	CV/10:30	25	54	47	CV/14:10	AG/14:40	59
22	17	CV/10:00	AG/10:30	26	55	50	AG/14:20	CV/14:50	62
23	19	CV/10:15	AG/10:45	0	56	49	CV/14:20	AG/14:50	61
24	20	AG/10:20	CV/10:50	27	57	52	AG/14:30	CV/15:00	65
25	21	CV/10:30	AG/11:00	28	58	51	CV/14:30	AG/15:00	63
26	22	AG/10:40	CV/11:10	29	59	54	AG/14:40	CV/15:10	0
27	24	CV/10:50	AG/11:20	30	60	53	CV/14:40	AG/15:10	0
28	25	AG/11:00	CV/11:30	31	61	56	AG/14:50	CV/15:20	65
29	26	CV/11:10	AG/11:40	32	62	55	CV/14:50	AG/15:20	0
30	27	AG/11:20	CV/11:50	33	63	58	AG/15:00	CV/15:30	0
31	28	CV/11:30	AG/12:00	34	64	57	CV/15:00	AG/15:30	0
32	29	AG/11:40	CV/12:10	35	65	61	CV/15:20	AG/15:50	0
33	30	CV/11:50	AG/12:20	36					

Note: "AG" stands for Apple Grove, "CV" stands for Centreville.

part of a vehicle schedule which begins at period 13 (the bus leaves the garage at 13:10 to arrive at Centreville at 13:20) and ends at period 15 (trip 65, which arrives at Apple Grove at 15:50 and at the garage at 16:00). The start and end times of these blocks are easily found by "threading through" the predecessor and successor trips in Table 5.1.

If we hook trip 38 with trip 44 instead of trip 43, we incur a layover of ten minutes but we create a vehicle schedule solution which "costs out" optimally in terms of the HM2 model. Now trips 38 and 44 form part of a new vehicle schedule which begins at period 8 and ends at period 15, and trip 43 begins a block in period 13 (leaving the garage at 13:10 to depart Centreville at 13:20) and ends at period 14. This is exactly the change to the VSP2 solution which seemed to be called for in Table 4.6.

Thus we have found a way to reduce estimated manpower cost in our solution by 2.4% while increasing platform time by only ten minutes (less than .5%).

5.4 A heuristic for creating efficient vehicle schedules

This section describes a heuristic for solving the vehicle schedule improvement problem posed in Section 5.2. The heuristic is based principally upon the observation that in general there are many sets of y_{qr} which give an optimal solution to HM2, and that one of these sets may resemble to a large extent the set of y_{qr} values which models the VSP2 vehicle solution. The heuristic first "moves" the HM2 optimum "toward" the VSP2 solution by identifying those y_{qr} which can be forced into the HM2 basis, at the values which model the VSP2 solution, without increasing the value of the objective function. While these values can be found exactly be computing penalty values, as is discussed in Chapter 8, a good indication of an alternate

optimum is given by the presence of nonbasic y_{qr} with zero reduced costs.

This search of alternate optima yields a group of y_{qr} values which is both part of a HM2 optimum and which model part of an existing feasible set of vehicle schedules (see Figure 5.3). The y_{qr} which represent existing vehicle schedules but which do <u>not</u> enter the HM2 optimum are interesting, because in a sense the HM2 model "rejects" them. In other words, if any of these "rejected" y_{qr} were to enter the HM2 basis, it would force up the value of the objective function and hence imply higher manpower cost to cover the feasible block which it represents. The "rejected" y_{qr} indicate vehicle schedules which should be eliminated, through rehooking, in the feasible vehicle schedule solution.

The next step in the heuristic is to make just those rehookings, but without creating others which the HM2 model "rejects". Thus the heuristic seeks to always increase the y_{qr} which are common to an optimal HM2 solution and to a set which models an existing vehicle schedule solution.

The heuristic is iterative because a new feasible vehicle solution may give rise to a new opportunity to "move" the HM2 optimum to another which further increases the overlap between the two sets of y_{qr}. Thus the heuristic alternates between seeking alternate HM2 optima and seeking alternate feasible vehicle solutions.

The heuristic terminates when one of two conditions is met: (a) a feasible vehicle solution is found which "costs out" optimally in the HM2 model, or (b) we can find no alternative HM2 optimum which more closely models the existing feasible solution <u>and</u> we can find no rehookings which would eliminate "rejected" vehicle schedules without creating others.

Figure 5.4 is a flow chart of the heuristic.

This heuristic has the advantage that it can be carried out using

Figure 5.3

Reconciling an HM2 optimum with a feasible vehicle solution

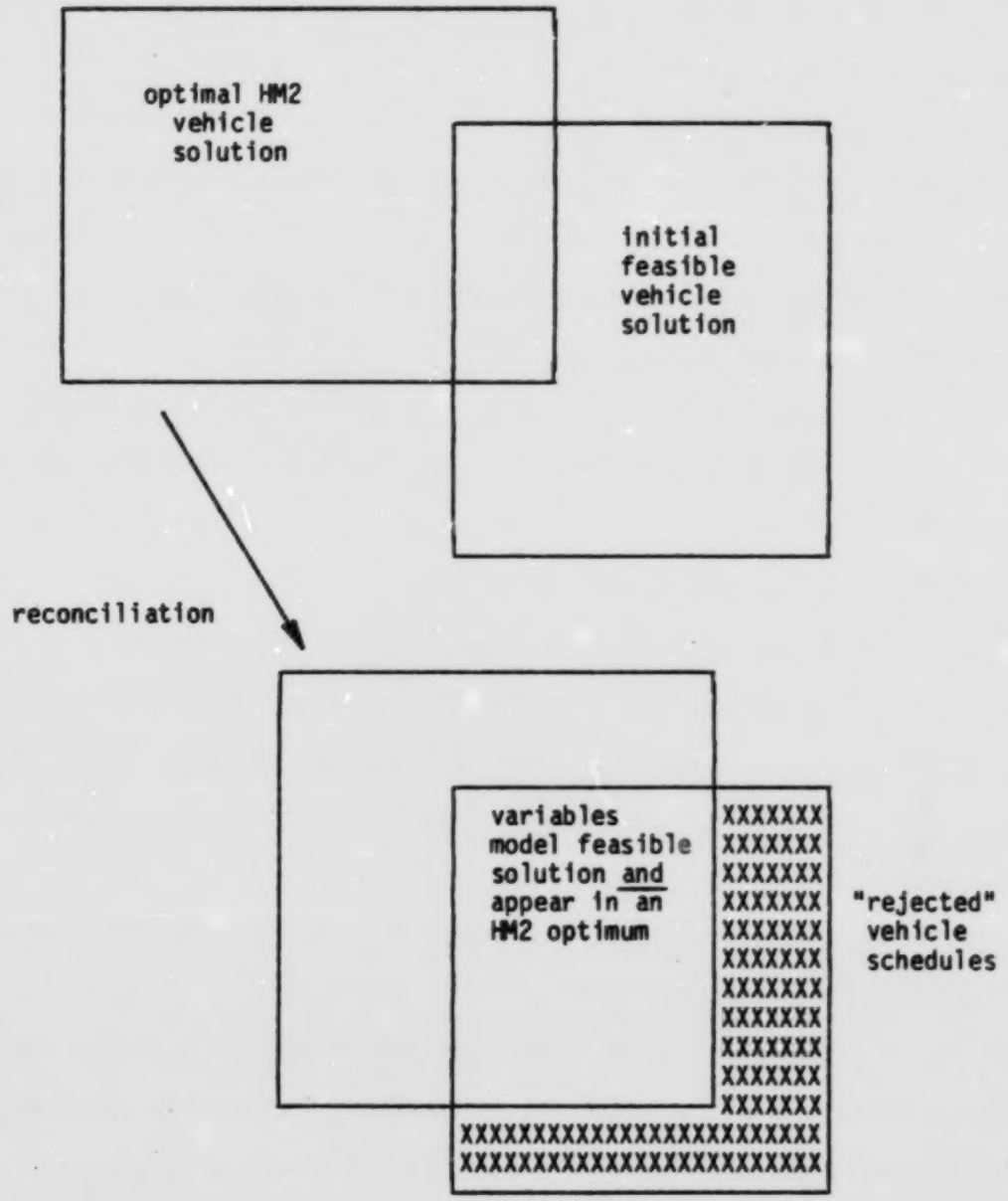
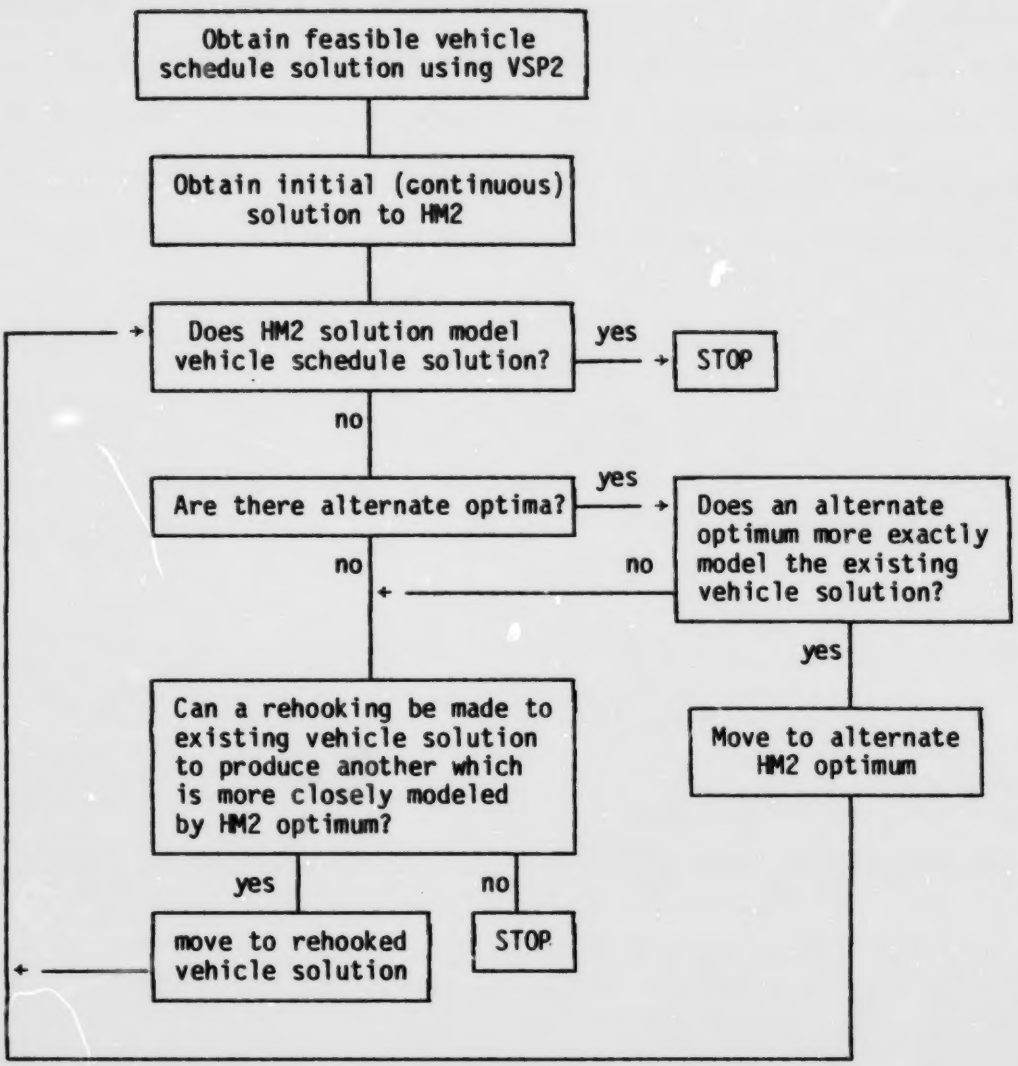

optimal HM2 vehicle solution
initial feasible vehicle solution
reconciliation
variables model feasible solution and appear in an HM2 optimum
"rejected" vehicle schedules

Figure 5.4

Flowchart of vehicle schedule improvement heuristic


Obtain feasible vehicle schedule solution using VSP2
Obtain initial (continuous) solution to HM2
Does HM2 solution model vehicle schedule solution?
yes
STOP
no
Are there alternate optima?
yes
Does an alternate optimum more exactly model the existing vehicle solution?
no
no
yes
Can a rehooking be made to existing vehicle solution to produce another which is more closely modeled by HM2 optimum?
Move to alternate HM2 optimum
yes
no
move to rehooked vehicle solution
STOP

existing linear programming solution packages, since the analyst is only required to fix certain variables at preassigned values and reoptimise the linear program. It has the disadvantage, however, that it requires several reoptimisations of the same model in the course of a single iteration and hence that it may take a long time. Another is that it does not guarantee an optimal solution because it is not a thorough search for a "best" set of vehicle schedules. Despite these limitation, we will see in the next chapter that the heuristic is capable of producing positive results.

Chapter 6

A demonstration of the vehicle scheduling method using trip timetable data from Montréal

6.1 Introduction

This chapter demonstrates the vehicle scheduling method of this research using trip data found in actual trip timetables. The transit service in these trip schedules was offered in early 1983 by the transit authority in Montréal, Québec (la Commission de transport de la Communauté urbaine de Montréal). We consider two transit lines, line 32 (Lacordaire) and line 139 (Pie-IX), both of which serve the east end of the City of Montréal and two adjacent suburbs. Both lines operate in a north-south direction and make connections with the underground "Metro" where passangers can transfer to rapid transit service into downtown Montréal.

Table D-1 (in Appendix D) gives the trip data itself for line 32, and Table D-2 that for line 139. Each of the entries in these tables gives a trip number, a trip start time, a start location, an end time, and an end location. At the bottom of each of these tables are deadhead times to and from the garage. In accordance with the current practices of the CTCUM we use a single bus garage, that in the suburb of Ville d'Anjou. We are not permitting any deadheading in this demonstration except for that to and from the garage. This prohibition simplifies the presentation of the results and conforms in large part to the vehicle scheduling practices of the CTCUM. Finally, we respect a ten-minute vehicle preparation and check-in requirement at the beginning and end of each vehicle schedule. This means that every block includes an extra ten minutes at its beginning (before the vehicle pulls out

of the garage) and again at its end (after the vehicle pulls in).

6.2 The initial vehicle schedule solution

The initial vehicle solution is found straightforwardly using the VSP2 model presented in Chapter 2. Table B-1 (in Appendix B) provides a list of block start and end times, including garage deadheads, for the VSP2 solution for line 32 and Table B-2 gives the same information for line 139. Successor and predecessor trips from this solution are noted in each of Tables D-1 and D-2.

The next step in the method is the discretization of the VSP2 solution. Because the manpower scheduling parameters of the CTCUM can be expressed in multiples of 30 minutes, this demonstration uses such a period length. We then calculate the start and end periods for each of the vehicle schedules found using VSP2 and report these in Tables B-1 and B-1. Figures B-1 and B-2 give the service histograms which arise from the discrete schedules. We note at this point the peaking characteristics of the two lines. On line 32, there are 21 vehicles in service at the highest peak and 9 in service during the trough in the middle of the day. Thus the peak-to-base ratio is 21/9 or 2.33 . For line 139, there are 35 vehicles in service at the highest peak and 13 at the lowest point during the day, which yields a peak-to-base ratio of 35/13 , or 2.69 .

Once the vehicle schedules have been discretized we can form the sets K and L of potential vehicle schedule start and end times. We have noted the composition of these sets at the bottom of Tables B-1 and B-2. We also use the sets K and L to represent the discretized VSP2 solutions in tableau format in Figures B-3 and B-4.

6.3 The HM2 problem matrix

Using the sets K and L, we generate the HM2 problem matrix. Since the HM2 problem matrix is based on the HM1 matrix used in the HASTUS-Macro manpower scheduling program, we take advantage of the existing software to create an initial problem matrix which we then augment with additional rows and columns.

Recall that Chapter 3 described how there are generally more small-block constraint rows in the HM2 problem matrix than in that of HM1 owing to the fact that we have to allow for all possible small blocks. Moreover, certain manpower-scheduling variables must have nonzero entries in these potential "small-block" constraint rows. In order to induce the existing software to produce the necessary rows and column variables, we use the device of creating a dummy block data set comprising exactly one vehicle schedule from period q to period r for each $q \in K$ and $r \in L$ where $q < r$. This device assures us that the HM2 matrix will be flexible enough to allow for all of the vehicle schedule possibilities contained in B and still include the appropriate feasibility constraints. Of course, this device causes the existing program to create a set of constraint constants which is of no value; we simply discard it.

The columns representing $x_{ij,mn}$ variables are formed using parameters which arise from the collective agreement in force at a particular transit property and from certain nonwritten rules and procedures which traditionally govern manpower scheduling. A list of the rules used in this research to process the CTCUM data is included in Table B-3. For simplicity in this demonstration, we are keeping the maximum number of pieces of work per duty at 2, even though 3-piece duties are allowed at the CTCUM. This restriction has the effect of reducing the size of the problem matrix considerably. The

remaining entries in Table B-3 conform to the rules in force at the CTCUM.

These parameters are presented to HASTUS-Macro in the form of a card-image file called a "Macro file". The Macro file used for analysing the CTCUM data is presented in Table B-4. A full description of all HASTUS-Macro commands can be found in GIRO, Inc. (1983). We note, however, that the definition of a "small block" is found in the HASTUS-Macro commands through the use of the MAX-FEASIBILITY parameter, which gives the value of the maximum length of a "small block", in hours. The ratio

$$\frac{\text{MAX-FEASIBILITY}}{\text{PERIOD}}$$

is in fact the value of the parameter W of Chapters 3 and 4.

The HASTUS-Macro program then proceeds to produce a large card-image file in MPS format (Control Data Corportation, 1979). Because the input vehicle schedule solution is in fact dummy data representing all possible potential vehicle schedules, there will be somewhat more rows and columns represented in this MPS file than would normally be the case for a HASTUS-Macro run. We discard the vector of constraint constants in this file and add additional row records representing the service covering constraints and additional column records corresponding to the y_{qr} variables, in accordance with the matrix construction rules of Table 4.3.

6.4 Solving the HM2 linear program

We then proceed to solve the linear program using standard linear programming code, in the case of this research the APEX-III package (Control Data Corporation, 1979). Tables B-5 and B-6 give some summary statistics on the solution of the HM2 linear program for the two lines under study. At the

bottom of each of Tables B-5 and B-6 are two values of the objective function for the HM2 linear program. The first of these is that achieved at the optimum. The second is that which is obtained by fixing the values of the y_{qr} at those of Figures B-3 and B-4. We see that for both lines the two values are very similar, in fact are seprated by a difference of .50% for line 32 and .57% for line 139. Since these figures express an estimate of the total difference between the cost of serving a set of "optimal" vehicle schedules and that of serving the VSP2 vehicle schedules, we can estimate a total margin of potential cost improvement at somewhere around .5% for each line.

Figures B-5 and B-6 present the y_{qr} variables from the optimal solution in tableau format. We see that the solutions are heavily noninteger, but that the rows and columns of these tableaux sum to the constants C_r and R_q as discussed in Section 3.5. At this point we can compare the continuous HM2 solution and the set of y_{qr} which models the VSP2 solution for each of the two lines under study. We have underlined, in Figures B-5 and B-6, those entries which also appear (perhaps with different magnitude) in Figures B-3 and B-4. This gives an indication of the degree to which the discrete VSP2 solution and the obtained continuous HM2 optimum agree. We see that there are comparatively few y_{qr} variables which are nonzero in both solutions, especially in the case of line 32.

We include, at the top of each of the tableaux of y_{qr} values in Appendix B, the HM2 objective function value which would result from optimizing HM2 with the y_{qr} fixed at the values in the tableau. This is the case as well for Figures B-3 and B-4, which represent the discrete VSP2 solution.

6.5 Applying the vehicle schedule improvement heuristic

Following the development of Section 5.4, we apply the vehicle schedule improvement heuristic to the data of the two lines under study. We first seek alternate optimal solutions to HM2 by investigating the reduced costs associated with nonbasic y_{qr} in the obtained optimum. Figures B-7 and B-8 present these reduced costs, again in tableau form. The squares marked with an asterisk represent those y_{qr} which are not in the basis but for which the reduced cost is zero. Those marked with 'b' are basis in the optimal solution, and those marked with 'n' are basic at zero in that solution. Again, we underline non-zero entries in squares corresponding to those y_{qr} which model the VSP2 solution. The underlined asterisks are interesting in the context of the improvement heuristic because they indicate the possibility of introducing y_{qr} associated with the VSP2 solution into the HM2 basis without increasing the value of the objective function and thus of finding an alternate optimum which more closely models the VSP2 solution.

Figures B-9 and B-10 present sets of y_{qr} variables which have the property that each induces an optimal solution to HM2 and each comprises a large number of y_{qr} implied by the VSP2 solution, but that we cannot force any other such y_{qr} into the basis without increasing the value of the objective function. We note with an "R" the "rejected" y_{qr}, that is, those which represent feasible blocks in the VSP2 solution but which cannot be introduced into the basis at their VSP2 value without increasing the value of the objective function. These are the vehicle schedules to be eliminated, if possible, by rehooking pairs of trips in the feasible vehicle schedule solution.

Note in these tableaux that <u>any</u> VSP2 block of three periods' duration or less is rejected. This follows directly from the fact that we are constrained to pay a piece guarantee on any piece of work less than two hours (four

periods) in length. Note that the equality constraints (3.16) will force several longer vehicle schedules to be "rejected" at the same time as the shorter ones, because of the limited degrees of freedom on the y_{qr} in any tableau to vary. One physical interpretation of this phenomenon, in fact, is that we always have to rehook vehicle schedules in pairs -- to make a shorter vehicle schedule longer, we have to make a longer one shorter. We can conclude, however, that there are probably far fewer truly problematic vehicle schedules than there are "rejected" squares in either of Figure B-9 or B-10.

Table B-6 summarizes the improvement heuristic applied to the data of line 32 and Table B-7 does the same for the data of line 139. In these tables a "basis change" indicates a move to fix a y_{qr} at the value it takes in the discrete VSP2 solution, without modifying at all the feasible vehicle solution (this represents the path on the right hand side of Figure 5.4). A "rehooking" indicates a change to the feasible vehicle solution as described in Section 5.2. Changes to successor and predecessor trips in Tables D-1 and D-2 are listed for each rehooking in these tables. We also note the change in the vehicle schedule configuration associated with each rehooking. For example, in step 2 of Table B-6,

$$\begin{bmatrix}14\text{-}16\\10\text{-}20\end{bmatrix} \rightarrow \begin{bmatrix}10\text{-}16\\14\text{-}20\end{bmatrix}$$

means that before the rehooking there was a feasible vehicle schedule from period 14 to period 16 and another from period 10 to period 20, and afterwards one from period 10 to period 16 and another from period 14 to period 20. Note in the case of rehookings that we do not take "jumps" over period boundaries into account in assigning y_{qr} variable values but simply switch start and end periods. We do, however, keep track of the number of minutes added to the

vehicle solution by each rehooking.

In the case of line 32 we are able to produce a vehicle solution which is in fact modelled by an optimal set of y_{qr}, which is given in Figure B-11. In the case of line 139, on the other hand, we are not able to do so (see Figure B-12): The value of the objective function associated with the "improved" vehicle solution in this case is .82 hours above that at the continuous optimum. We compute a "total cost increase" for the heuristic by adding the objective function change to the total hours (minutes divided by 60) added to the vehicle solution. (Recall that the objective function is in units of hours.) As Section 5.4 described, the additional vehicle hours are included in this figure to acknowledge that we are in fact adding vehicle time to the solution during the heuristic and to compensate for the lack of attention to "jumps" over period boundaries. The final figure in these tables gives an indication of how much of the estimated potential improvement (the "difference" figure from Tables B-5(a) and (b)) has been recuperated during the course of the heuristic. We note that the heuristic has succeeded in keeping about half of the estimated potential improvement in the case of line 139, and almost all of it in the case of line 32.

A summary of the final vehicle solution after applying the vehicle schedule improvement heuristic is presented in Table B-8 for line 32 and in Table B-9 for line 139.

6.6 Creating actual manpower assignments

We create feasible manpower assignments using the HASTUS program (Rousseau, Lessard, and Blais, 1982) which has been calibrated for the scheduling problem of the CTCUM. The first step in constructing assignments is the extraction of relief points. We described in Chapter 1 how relief points and relief times relate to vehicle schedules and gave an example in Figure 1.1. Table B-10 gives the information on the position of the relief point on each of lines 32 and 139. The running time to the relief point depends on the total running time between termini. For the range of possible running times on each of the two lines, the appropriate time to the relief point is given in these tables.

HASTUS is heavily parameterized and any solution will depend on the parameter values chosen by the analyst. The first such set of parameters to consider is the Macro file itself. The Macro file used in this manpower scheduling exercise is reported in Table B-11(a). The period length is 34 minutes in this file, instead of 30 minutes as it was for generating the HM2 problem matrix earlier in the chapter. The CTCUM uses this period length in their manpower scheduling, because it most satisfactorily represents their particular scheduling conditions. For a full explanation of each of the parameters in a Macro file see GIRO, Inc. (1983).

The second set of parameters is contained in what is known as a "selection file". This file is used to penalize assignments of certain types. These penalties enter into the cost calculation when final optimization is carried out (Rousseau, Lessard, and Blais, 1983). The selection file used in this demonstration is reported in Table B-11(b). We penalize, with the maximum permitted penalty, any assignment with more than two pieces and any tripper, or short assignment which does not form a regular operator workday.

As Section 1.3 described, the use of trippers is generally inefficient and we wish to discourage their appearance in a final manpower solution. We doubly penalize any tripper which occurs either very early in the day or very late in the day, because the CTCUM does so for a variety of institutional reasons.

The final parameter which we need to set is that of the number of operators requested in a solution. The HASTUS system is structured in such a way that the system will never create more than a given number of regular workdays and will put into trippers any vehicle time which cannot be covered by these regular workdays. We determine this number in the following fashion. Let V be the total vehicle time in a set of vehicle schedules, and let L be the expected average length of a regular workday. The requested number of workdays is then taken to be

$$[V/L] + 1$$

where $[.]$ is the truncation operator, as in Chapter 4. Since the CTCUM is constrained to a maximum platform time of 8 hours, we set L equal to 7 hours and 50 minutes to reflect the inability of the system to meet the 8 hour constraint exactly for every assignment. For line 32, there are 249 hours and 3 minutes of vehicle time in the VSP2 solution and 249 hours and 14 minutes in the adjusted solution. Thus we choose 32 operators for line 32. For line 139 there are 399 hours and 4 minutes in the VSP2 solution and 399 hours and 26 minutes in the adjusted solution. Thus we choose 51 operators for this line.

The results of the manpower scheduling are summarized in Table B-12. We note, first of all, that none of the manpower solutions is without trippers, even though we have requested no trippers in any solution and we have penalized them heavily in the parameters. If we compare the cost of the manpower solutions using the adjusted vehicle schedules against that using the VSP2

vehicle schedules, we find that the total cost of the regular duties (excluding trippers) is lower for the adjusted vehicle solution on both lines. Since it is unrealistic to ignore trippers completely, however, we attempt to assign a cost to them by assuming that any tripper will be worked in overtime by a regular operator and thus paid at 1.5 times the regular hourly rate of 10.51. Once this additional cost is included in the calculations of Table B-12 we find that the differences cost to cover the two vehicle solutions on each line is negligible.

In reality, the CTCUM does not do manpower scheduling line-by-line and would schedule an entire division (encompassing several lines) at one time in any pass of the HASTUS system. By scheduling much smaller problems we have in fact biased the problem against ourselves. In addition, the CTCUM can use 3-piece duties, which might have helped to improve the solutions had we done so as well. The fact remains, however, that while we can demonstrate a savings in manpower cost on each of the two lines under study at the level of HASTUS-Macro, we are unable to do so at the level of feasible manpower assignments using the entire HASTUS system. This somewhat disheartening finding indicates that there is room for improvement of the HM2 model and the heuristic procedure. We present some possible directions for future research in Chapter 8.

6.7 Estimating potential improvement under different scheduling conditions

In Section 6.4 we estimated, using the HM2 model, the overall potential for improvement of the VSP2 solution in the case of either line 32 or line 139 at about .5%. Since this value is quite small, we calculate this estimate as well using scheduling constraints different from those in Table B-3 to see if perhaps different constraints would indicate a large potential for improvement. We estimate the potential for improvement under each of four senarios:

a. A reduction of the maximum spread from 13 hours to 11 hours and 30 minutes;
b. A change, from 12 hours to 11 hours, in the limit after which a full-hour premium is paid on spread;
c. A change in the piece guarantee from two hours to three hours; and
d. A change in the minimum percentage of straight duties from 25% to 35%.

Table B-13 contains the values of the HM2 objective function with the y_{qr} both free to vary and fixed at the discrete VSP2 solution, and the difference between these two values, for line 32 and line 139. This difference is the estimated potential improvement under each of the scenarios. We see that the potential remains small in all cases, never rising above 1% except for an increased piece guarantee. In fact, for each of the changes in the spread premium the potential improvement is estimated as being even smaller than that under the constraints of Table B-3.

The columns marked '%Δ' give the percentage increase in the objective function associated with each change in the scheduling constraints, for both free and fixed y_{qr} and for each line under study. Interestingly, decreasing

the maximum spread forces up the value of the HM2 objective function quite substantially in the case of line 139 but not in the case of line 32. Note, however, that despite this difference the potential for improvement is estimated as being about the same.

Chapter 7

Using HM2 to estimate the marginal costs of transit service

7.1 The dual of the HM2 linear program

Because the HM2 model is a linear program, there is a second, equivalent way to pose the optimization problem (Dantzig, 1963). This second problem, called the dual linear program, is one of constrained maximization. The objective function to be maximized is a linear combination of quantities called dual variables, one of which is associated with each constraint in the original, or primal, linear program. The coefficients in this linear combination are the elements of the right-hand side of the primal linear program. The constraints on maximization are formed from the transpose of the problem matrix of the primal linear program, and the right-hand side vector for the new constraint set is the cost vector in the primal linear program. One of the properties of a dual linear program is the fact that the value of its objective function at its constrained optimum is the same as the value of the value of the primal objective function at its constrained optimum (Dantzig, 1963).

We can state the dual of the HM2 linear program as follows:

$$\text{Maximize} \quad \sum_{p \epsilon P} s_p \rho_p + \sum_{f \epsilon F} q_f \omega_f \qquad (7.1)$$

$$\text{subject to} \quad \sum_{p=i}^{j} \beta_p + \sum_{p=m}^{n} \beta_p + \delta_{ij}\sigma_{ij} + \delta_{mn}\sigma_{mn} + \delta_m \pi_m$$

$$+ \sum_{f=1}^{F} \delta_{f,ijmn}\omega_f \leq c_{ij,mn} \ , \ (ij,mn) \in D \qquad (7.2)$$

$$\sum_{p=q}^{r} (\rho_p - \beta_p) - \delta_q \pi_q - \delta_{qr}\sigma_{qr} \leq 0 \ , \ (qr) \in B \qquad (7.3)$$

where the β_p , $p \in P$ are the nonnegative dual variables associated with the block-covering constraints (3.9), the π_q , $q \in K$ those associated with the pull-out constraints (3.11), the σ_{qr} , $(qr) \in S$ those associated with the small-block constraints (3.10), the ρ_p , $p \in P$ those associated with the service-covering constraints (3.12), and the ω_f , $f \in F$ those associated with the supplemental constraints (3.5). The single-subscripted δ_q is a Kronecker delta equal to 1 if $q \in K$ and to 0 otherwise. The double-subscripted δ_{qr} is another Kronecker delta equal to 1 if $(qr) \in S$ and to 0 otherwise Finally, the multi-subscripted $\delta_{f,ijmn}$ is a Kronecker delta which is equal to 1 if $(ij,mn) \in C_f$ and to 0 otherwise. The s_p for $p \in P$ and the q_f for $f \in F$ are the constants on the right hand side of (3.12) and (3.15), respectively.

For the sake of consistency, we are replacing the equality constraints (3.13) by the inequality constraints (3.12) in this exposition, even though we have used (3.13) exclusively up to this point. The use of (3.13) in the primal HM2 linear program means, strictly speaking, that the ρ_p are unrestricted in sign in the dual HM2 linear program. The fact that we can replace (3.12) with (3.13) in the primal problem without affecting the value of the objective function implies in effect that the nonnegativity constraints on the

ρ_p are redundant in the dual linear program, inasmuch as an increase in the row activity in any of the service-covering constraints in the primal problem will never reduce the value of the objective function in that problem.

Note that only the ρ_p and the ω_f enter into the objective function (7.1). This is because the right hand side is zero in (3.9), (3.10), and (3.11).

An interesting aspect of dual linear programs is the way in which the solution to a dual problem can be interpreted. According to theory (Dantzig, 1963; Dorfman, Samuelson, and Solow, 1958) the value of a dual variable at optimum can be seen as an estimate of the change in the total value of the objective function which would be associated with a change in its associated constraint by one unit. In particular, each ρ_p has an interpretation as the marginal cost (or saving), in terms of total labor cost, associated with an increase (or decrease) of one unit of the corresponding s_p. But since each s_p is a required level of service in the p^{th} period, we are led to interpret ρ_p as the marginal labor cost or savings associated with a change in the amount of service at period p, for all $p \in P$.

The interpretation of the β_p, the σ_{qr}, and the π_q of (7.1) - (7.3) is not as straightforward. The constraints (3.9) - (3.11) with which these variables are associated in fact serve to relate the $x_{ij,mn}$ and the y_{qr} variables within the HM2 problem. As such they can be interpreted as marginal manpower costs associated with a particular optimal solution in the y_{qr}. The variable β_p would thus be treated as the marginal general labor cost associated with covering the optimal y_{qr} solution in period p. Similarly, the π_q and σ_{qr} could be interpreted as the marginal labor cost associated with covering an additional pull-out at period q or a small block from pe-

riod q to r, respectively, in the region of the optimal solution in the y_{qr}. In any event, we note that that the relationships between the ρ_p and the β_p, π_p, and σ_{qr} are defined by the dual constraints (7.3), which can be expressed as

$$\sum_{p=q}^{r} (\rho_p - \beta_p) \leq \delta_q \pi_q + \delta_{qr} + \sigma_{qr}, \quad (qr) \in B$$

Thus in a general sense we should expect the ρ_p and the β_p to be similar in magnitude.

7.2 Obtaining a unique dual solution

An important technical problem arises in connection with the uniqueness of the solution to the dual of any particular realization of HM2. It is quite possible that the primal HM2 linear program will be degenerate at the optimum, that is, that an optimal solution to the primal HM2 problem will include many basic variables with values of zero. What this can mean is that there may be several alternate optimal solutions to the dual problem. We note, however, that since each of these optimal solutions is an extreme point of the feasible region to the dual problem (Charnes, Cooper, and Henderson, 1953), these alternate optimal solutions form a subset of the extreme points in this region. As such they form a convex set. We will call this convex set the "optimal region" of the dual HM2 linear program.

To simplify the exposition in this section somewhat, we introduce some additional notation. Let

$$\phi^T = (\beta^T, \pi^T, \sigma^T, \omega^T, \rho^T)$$

where β^T, π^T, σ^T, ω^T, and ρ^T are row vectors of the β_p, π_q, σ_{qr}, ω_f

and ρ_p respectively. Thus ϕ is simply a convenient way to represent the entire vector of dual variables at the same time.

We will denote by Ψ the optimal region of the dual of HM2. If there are m dual variables, Ψ is a subset of the nonnegative orthant of m-dimensional real variable space R^m.

If Ψ has nonzero dimensionality, that is, there is more than one alternate optimal solution to the dual of HM2, then strictly speaking there are an infinite number of alternate optimal solutions to the dual problem. While it does not necessarity follow from this that any one solution is totally without meaning, it could still be true that some dual solutions are more interesting than others. Unless we can know all the optimal solutions to such a linear program, however, we are never in a position to choose between alternate solutions.

Rather than try to enumerate alternate optimal solutions, this research locates a single point $\phi \in \Psi$ which satisfies certain properties. First, we choose a candidate point ϕ^* which may or may not be an element of Ψ. This point ϕ^* could be interpreted as representing a noninformative or conservative solution to the dual of HM2, if it were in fact an element of Ψ. We then proceed to find the element $\phi' \in \Psi$ which is closest to ϕ^* in a Euclidean metric.

This problem can be stated as follows. Let ϕ^* denote an arbitrary fixed point in the nonnegative orthant of R^m, and let ϕ denote a variable point in this same subspace. Let z^* be the value of the optimal solution to HM2. We want to minimize

$$(\phi - \phi^*)^T(\phi - \phi^*) \qquad (7.4)$$

subject to

$$\sum_{p \in P} s_p \rho_p + \sum_{f \in F} q_f \omega_f = z^* \qquad (7.5)$$

to (7.2), to (7.3), and to nonnegativity in all variables. The form (7.4) is a quadratic function in the vector ϕ, and hence the problem is one of minimizing a quadratic function subject to linear constraints.

The noninformative point we choose to be our ϕ^* is one with all elements equal to zero except the ρ_p^* which are set equal and proportional to the rate of remuneration for one period's work, based on the operators' collective agreement. Our ϕ^* is conservative in the sense that it proposes that each ρ_p be equal, without further knowledge to the contrary, to nothing more than what would have to be paid an operator to cover period p during the course of a regular duty which involved no premium payments. It proposes as well that all other marginal costs be set to zero. Thus the quadratic program is used to find the optimal solution which is closest, in a Euclidean sense, to a solution which corresponds to strictly traditional costing methods using average costs. Our solution contributes information about transit costs, then only to the extent that the traditional procedure is inconsistent with the costing problem as modelled by HM2.

7.3 An application of the method to the CTCUM data

We solve the quadratic program of Section 7.2 using the same data which were used in Chapter 6 to demonstrate the vehicle scheduling method of this research. Now however, we extend the period length from 34 to 60 minutes. We

do this to reduce the size and complexity of the dual HM2 linear program, and thereby be able to take advantage of existing nonlinear programming code to solve the quadratic program.

The new period length gives rise to a new histogram of the VSP2 vehicle scheduling solution for each of lines 32 and 139. These histograms are presented in Figure B-13. Note in the case of line 139 that the "trough" at mid-day is at 14 vehicles rather than 13 as is the case in Figure B-2. What is hidden in this histogram is a vehicle schedule which ends in period 10 and another which begins in period 11. Thus we add a "pull-out" constraint of the form (3.4) for period 11.

Because of the change in period length, we have to change the Macro file as well. The new Macro file is presented in Table B-14. We represent as faithfully as we can the manpower scheduling constraints of Table B-3 while acknowledging the decreased flexibility with which the longer period length encumbers us. There are only two changes not related to the new discretization: that of reducing the MAX-FEASIBILITY parameter from six hours to four hours and that of allowing columns representing trippers. The first change more accurately relates MAX-FEASIBILITY to the piece guarantee (the ratio should be 2:1) and the second makes up for the long period length by generating a somewhat more realistic and flexible cost model.

We generate the HM2 problem matrices in the same way as we did in Chapter 6, but for the analyses of this chapter we use the MINOS mathematical programming package (Murtagh and Saunders, 1981) which can be used to solve either linear or nonlinear programs. Solution summaries for the primal HM2 solution are available in Table B-15(a).

Table B-16 contains the values of the dual variables from the solution of HM2, using hour periods, for the data from line 32 and Table B-17 those for

line 139. The first column contains those dual variables which were obtained from solving the primal HM2 problem. Paying particular attention to the service-covering constraint rows, we see that the HM2 primal solution has given us a set of dual variables which are fairly regular for the morning peak and mid-day periods but which include both zero values and elevated values for the afternoon peak and evening periods. We also note in passing that for both lines the dual variable associated with the constraint that 25% of all duties be one-piece is zero, as are those associated with all small-block and pull-out constraints.

We now apply the quadratic programming procedure of Section 7.2. For each line, we transform the primal MPS file into another representing the dual problem, and add the constraint (7.5). We again use the MINOS program to solve the quadratic program with objective function (7.4). Solution summaries for the quadratic program are contained in Table B-15(b). We take advantage of a starting basis for the quadratic program which is derived from the final basis in the primal problem (using the DUMP/LOAD feature of MINOS) and arrive very quickly at feasible and optimal solutions to the nonlinear problem.

The dual variables at the optimum of the quadratic program are found in the second column of Table B-16 for line 32 and of Table B-17 for line 139. We note that the quadratic adjustment has had the effect of smoothing the distribution of dual variables somewhat, although the raw and adjusted solutions are for the most part very similar to one another.

Interestingly, the dual variables found by the quadratic program take on elevated values at times of the day other than during the peak periods. This phenomenon is particularly pronounced in the evening hours. We recall, however, from Table B-3 that Montréal places a very restrictive spread limit on pieces of work which finish late in the evening. In particular, duties which

finish after 2:00 a.m. may not be spread over more than ten hours. The dual variable solution may in fact be an indication that it is generally difficult to assign work during the evening peak to an operator who would also be working very late at night, and therefore that there are some natural diseconomies associated with late-night transit service.

Chapter 8

Conclusion and Recommendations

This research has extended the method of the HASTUS transit manpower scheduling system (Rousseau, Lessard, and Blais, 1983) to the scheduling of vehicles in public transit in an attempt to create vehicle schedules which are efficient in their use of manpower. The method has been to find an initial vehicle schedule solution using conventional techniques and then to improve this solution using a heuristic technique which takes account of the estimated manpower cost associated with alternate vehicle solutions. The cost model used in this research is a large linear program which generalizes that used in the HASTUS system as a relaxation of the transit manpower scheduling problem.

We found in Chapter 6 that the general approach developed in this research shows promise when applied to the scheduling of trips in two actual trip timetables available from the Montréal, Québec transit agency. The estimated manpower cost savings over traditional scheduling methods is modest but apparent (less than 1%) according to the estimation model itself. When we attempt to create feasible manpower assignments from the improved vehicle solutions using existing manpower scheduling techniques, however, we are not able to show that the new vehicle scheduling method has any clearcut advantages over existing methods.

The estimated savings of Chapter 6, while small, could still translate into significant dollar savings if in fact they can be realized in practice. The 88 United States transit systems with 100 or more revenue vehicles report heving paid vehicle operators' wages totalling approximately 1.5 billion dollars in the 1981-1982 fiscal year (Transportation Systems Center, 1983).

.5% of that figure amounts to 7.5 million dollars. If fringe benefits and administrative costs were included as well, the estimated dollar savings would be even higher.

While this research has posed the problem of creating efficient vehicle schedules using a linear programming relaxation and has worked the problem through in a first pass, there is clearly a need for further development of the methodology before it becomes fully serviceable. First, the HM2 estimation model itself should be enhanced, if possible, to take into account further constraints imposed on a manpower solution by a vehicle solution. For example, the present model only includes constraints on pull-out times (many of which are effectively redundant) and on the existence of pieces of work in a manpower solution which conform to short vehicle schedules. The full HASTUS system, however, takes into account the full pattern of relief times in a vehicle solution when formulating a HASTUS-Macro relaxation, to ensure that one-piece duties of a certain length only appear in a solution to HASTUS-Macro if in fact they can be created from the existing vehicle solution (Blais, 1984). More constraints of this nature in the HM2 model might lead to more realistic cost estimates and hence to more satisfactory manpower solutions once vehicle schedules are created.

Second, the heuristic procedure of Chapter 5 should be enhanced to provide a more thorough search over feasible vehicle solutions. The "re-hooking" mechanism introduced in that chapter is a straightforward means to pass from one feasible solution to another and should pose minimal computational problems. In addition, known linear programming reoptimization techniques, such as the dual simplex method, could certainly be adapted to evaluate the efficiency of alternate vehicle solutions. An algorithm similar to the "branch and bound" technique of mixed-integer programming (Tomlin,

1970) would quite possibly be appropriate in this context.

Chapter 7 presented a method for obtaining a unique solution to the dual of HM2, which is often indeterminate because of degeneracy in the primal problem. The method involved minimizing a quadratic function subject to the original dual constraints and to a new constraint that any solution be optimal in the original problem. The HM2 dual variables have a natural interpretation as marginal costs associated with small changes in service levels during discrete periods of the day. We found that, in the case of the present data, such marginal costs were elevated during periods well outside of the service peaks, especially in the late evening hours. We observed that the existence of a spread premium and, in Montréal, particularly strict restrictions on the spread of duties which finish late in the evening may force up the cost of transit service at this time of the day. These findings indicate that there may be circumstances under which the use of a single average cost figure for off-peak transit service may understate the real cost implications of a service decision.

Appendix A

Glossary of technical transit terms

Assignment	see Duty.
Block	see Vehicle schedule.
Deadhead	A trip made by a transit vehicle while not in revenue service, usually to connect two revenue trips or to enter or leave the garage.
D-trip	In the approach of Ball, Bodin, and Dial (1983), the portion of a trip during which the vehicle and operator must stay together.
Duty	The work performed by a single operator in a single day.
Extraboard	A group of operators who stand by to perform irregular work or to relieve scheduled operators who for some reason are unable to perform their duties.
Hook	A link between two trip in a vehicle schedule.
Interlining	The practice of allowing trips on different lines to be pooled together in forming vehicle schedules.
Layover	Waiting time at a trip terminus between the end of one trip and the beginning of the next in a vehicle schedule.
Piece guarantee	The minimum amount paid for operating a piece of work, regardless of its actual length.
Piece of work	A single continuous period worked by an operator on a single vehicle.
Platform time	The total time to be covered by operators in a set of vehicle schedules.
Predecessor	The previous trip in a vehicle schedule.
Property	The agency which operates transit service.
Pull-in	The end of a vehicle schedule, where the vehicle returns to the garage.
Pull-out	The beginning of a vehicle schedule, where the vehicle leaves the garage.
Rehooking	A rearrangement of vehicle schedules whereby predecessors and successors of two or more trips are interchanged.

Relief point	The physical location where an operator can be relieved, so that a piece of work can begin or end.
Relief time	A time during the course of a vehicle schedule when the vehicle passes a relief point.
Rostering	The arrangement of duties into weekly assignments for operators.
Runcutting	The process of generating operator assignments from a fixed vehicle solution.
Spread premium	Extra pay to an operator because the lapse of time between the beginning and end of his/her duty is more than a set limit.
Straight duty	A duty which does not contain an unpaid break.
Successor	The subsequent trip in a vehicle schedule.
Trip	The movement of a vehicle, in revenue service, in a single direction between two termini.
Trip schedule	A set of trips from which a set of vehicle schedules are to be formed.
Tripper	A leftover piece of work which is normally covered by an extraboard operator.
Two-piece duty	A duty which includes an unpaid break.
Vehicle schedule	A sequence of trips which are served by the same vehicle between a pull-out from the garage and a pull-in to the garage.

Appendix B

Tables and figures from the CTCUM scheduling demonstration

TABLE B-1

VSP2 solution summary, line 32

Block	Garage(Per)	Start Time/Loc	End Time/Loc	Garage(Per)	Vehicle time
1	3:58 (8)	4:25/210	8:20/210	8:46 (17)	4:48
2	4:13 (9)	4:40/210	8:50/210	9:16 (18)	5:03
3	4:27 (9)	4:54/210	18:35/209	18:58 (37)	14:31
4	4:41 (10)	5:08/210	18:51/209	19:14 (38)	14:33
5	4:49 (10)	5:16/210	21:41/209	22:04 (44)	17:15
6	4:56 (10)	5:23/210	9:35/210	10:01 (20)	5:05
7	5:04 (11)	5:31/210	8:12/210	8:38 (17)	3:34
8	5:16 (11)	5:43/210	8:36/210	9:02 (18)	3:46
9	5:22 (11)	5:49/210	18:20/209	18:43 (37)	13:21
10	5:35 (12)	6:02/210	9:37/209	10:00 (20)	4:25
11	5:47 (12)	6:14/210	18:43/209	19:06 (38)	13:19
12	5:59 (12)	6:26/210	10:07/209	10:30 (21)	4:31
13	6:29 (13)	6:56/210	8:28/210	8:54 (17)	2:25
14	6:50 (14)	7:15/209	8:00/210	8:26 (16)	1:36
15	6:58 (14)	7:23/209	16:18/209	16:41 (33)	9:43
16	7:06 (15)	7:31/209	10:29/209	10:52 (21)	3:46
17	7:14 (15)	7:39/209	9:07/209	9:30 (19)	2:16
18	7:22 (15)	7:47/209	17:59/209	18:22 (36)	11:00
19	7:30 (16)	7:55/209	18:10/209	18:33 (37)	11:03
20	11:12 (23)	11:37/209	26:28/209	26:51 (53)	15:39
21	12:03 (25)	12:28/209	18:30/209	18:53 (37)	6:50
22	13:51 (28)	14:18/210	25:54/209	26:17 (52)	12:26
23	14:08 (29)	14:35/210	26:11/209	26:34 (53)	12:26
24	14:23 (29)	14:50/210	22:28/209	22:51 (45)	8:28
25	14:25 (29)	14:50/209	25:37/209	26:00 (52)	11:35
26	14:31 (30)	14:58/210	25:05/209	25:28 (50)	10:57
27	14:35 (30)	15:02/210	15:48/209	16:11 (32)	1:36
28	14:42 (30)	15:09/210	18:55/209	19:18 (38)	4:36
29	14:46 (30)	15:11/209	22:09/209	22:32 (45)	7:46
30	14:50 (30)	15:17/210	16:03/209	16:26 (32)	1:36
31	14:53 (30)	15:20/210	19:03/209	19:26 (38)	4:33
32	15:01 (31)	15:28/210	19:13/209	19:36 (39)	4:35
				TOTAL VEHICLE TIME:	249:03

K = { 8,9,10,11,12,13,14,15,16,23,25,28,29,20,31 }

L = { 16,17,18,19,20,21,32,33,36,37,38,39,44,45,50,52,53 }

Note: The (per) column indicates discrete time period.

TABLE B-2

VSP2 Solution Summary, Line 139

Block	Garage(Per)	Start Time/Loc	End Time/Loc	Garage(Per)	Vehicle time
1	4:30 (10)	4:55/739	8:36/740	9:06 (18)	4:36
2	4:38 (10)	5:10/740	22:40/739	23:05 (46)	18:27
3	4:45 (10)	5:10/739	18:28/740	18:58 (37)	14:13
4	5:00 (11)	5:25/739	26:03/739	26:28 (52)	21:28
5	4:58 (10)	5:30/740	20:19/739	20:44 (41)	15:46
6	5:08 (11)	5:33/739	9:08/740	9:38 (19)	4:30
7	5:13 (11)	5:38/739	8:30/739	8:55 (17)	3:42
8	5:19 (11)	5:44/739	19:46/739	20:11 (40)	14:52
9	5:28 (11)	5:53/739	8:03/740	8:33 (17)	3:05
10	5:32 (12)	5:57/739	8:08/740	8:38 (17)	3:06
11	5:36 (12)	6:01/739	8:13/740	8:43 (17)	3:07
12	5:41 (12)	6:06/739	9:48/740	10:18 (20)	4:37
13	5:49 (12)	6:14/739	8:30/740	9:00 (18)	3:11
14	5:53 (12)	6:18/739	21:51/739	22.16 (44)	16.23
15	6:02 (13)	6:27/739	8:42/740	9:12 (18)	3:10
16	6:07 (13)	6:32/739	24:35/739	25:00 (50)	18:53
17	6:17 (13)	6:42/739	8:57/740	9:27 (18)	[illegible]10
18	6:23 (13)	6:48/739	25:04/739	25:29 (50)	19:06
19	6:26 (13)	6:51/739	8:22/739	8:47 (17)	2:21
20	6:32 (14)	6:57/739	18:46/740	19:16 (38)	12:44
21	6:37 (14)	7:02/739	9:15/740	9:45 (19)	3:08
22	6:39 (14)	7:04/739	21:10/739	21:35 (43)	14:56
23	6:41 (14)	7:06/739	7:51/740	8:21 (16)	1:40
24	6:46 (14)	7:11/739	7:57/740	8:27 (16)	1:41
25	6:54 (14)	7:19/739	25:21/739	25:46 (51)	18:52
26	6:47 (14)	7:19/740	8:49/740	9:19 (18)	2:32
27	7:00 (15)	7:25/739	26:30/739	26:55 (53)	19:55
28	7:05 (15)	7:30/739	8:16/740	8:46 (17)	1:41
29	7:11 (15)	7:36/739	8:22/740	8:52 (17)	1:41
30	7:16 (15)	7:41/739	8:27/740	8:57 (17)	1:41
31	7:25 (15)	7:50/739	8:34/740	9:04 (18)	1:39
32	7:31 (16)	7:56/739	18:08/740	18:38 (37)	11:07
33	10:39 (22)	11:04/730	25:52/739	26:17 (52)	15:38
34	11:06 (23)	11:31/739	19:37/739	20:02 (40)	8:56
35	11:40 (24)	12:05/739	18:42/739	19:07 (38)	7:27
36	12:14 (25)	12:39/739	18:21/740	18:51 (37)	6:37
37	13:50 (28)	14:22/740	18:29/739	18:54 (37)	5:04
38	14:07 (29)	14:39/740	17:58/740	18:28 (36)	4:21
39	14:35 (30)	15:07/740	22:10/739	22:35 (45)	8:00
40	14:55 (30)	15:20/739	17:45/740	18:15 (36)	3:20
41	14:45 (30)	15:17/740	18:36/740	19:06 (38)	4:21
42	14:51 (30)	15:23/740	18:41/740	19:11 (38)	4:20
43	15:09 (31)	15:34/739	18:50/739	19:15 (38)	4:06
44	15:01 (31)	15:33/740	18:51/740	19:21 (38)	4:20
45	15:16 (31)	15:41/739	18:06/740	18:36 (37)	3:20
46	15:08 (31)	15:40/740	18:55/740	19:25 (38)	4:17
47	15:26 (31)	15:51/739	18:15/740	18:45 (37)	3:19
48	15:18 (31)	15:50/740	18:18/739	18:43 (37)	3:25
49	15:25 (31)	15:57/740	21:31/739	21:56 (43)	6:31
50	15:38 (32)	16:10/740	25:40/739	26:05 (52)	10:27
51	16:04 (33)	16:36/740	19:03/739	19:28 (38)	3:24
52	16:15 (33)	16:47/740	19:12/739	19:37 (39)	3:22
53	16:25 (33)	16:57/740	19:24/739	19:49 (39)	3:24
54	16:35 (34)	17:07/740	26:15/739	26:40 (53)	10:05
				TOTAL VEHICLE TIME:	399:04

K = { 10,11,12,13,14,15,16,22,23,24,25,28,29,30,31,32,33,34 }

L = { 16,17,18,19,20,36,37,38,39,40,41,43,44,45,46,50,51,52,53 }

Note : The (per) column indicates discrete time period.

Figure B-1

VSP2 histogram using 30-minute periods, line 32

Period	Time		No.	
8	3:30- 3:59	a.m.	1	X
9	4:00- 4:29		3	XXX
10	4:30- 4:59		6	XXXXXX
11	5:00- 5:29		9	XXXXXXXXX
12	5:30- 5:59		12	XXXXXXXXXXXX
13	6:00- 6:29		13	XXXXXXXXXXXXX
14	6:30- 6:59		15	XXXXXXXXXXXXXXX
15	7:00- 7:29		18	XXXXXXXXXXXXXXXXXX
16	7:30- 7:59		19	XXXXXXXXXXXXXXXXXXX
17	8:00- 8:29		18	XXXXXXXXXXXXXXXXXX
18	8:30- 8:59		15	XXXXXXXXXXXXXXX
19	9:00- 9:29		13	XXXXXXXXXXXXX
20	9:30- 9:59		12	XXXXXXXXXXXX
21	10:00-10:29		10	XXXXXXXXXX
22	10:30-10:59		8	XXXXXXXX
23	11:00-11:29		9	XXXXXXXXX
24	11:30-11:59		9	XXXXXXXXX
25	12:00-12:29	p.m.	10	XXXXXXXXXX
26	12:30-12:59		10	XXXXXXXXXX
27	1:00- 1:29		10	XXXXXXXXXX
28	1:30- 1:59		11	XXXXXXXXXXX
29	2:00- 2:29		14	XXXXXXXXXXXXXX
30	2:30- 2:59		20	XXXXXXXXXXXXXXXXXXXX
31	3:00- 3:29		21	XXXXXXXXXXXXXXXXXXXXX
32	3:30- 3:59		21	XXXXXXXXXXXXXXXXXXXXX
33	4:00- 4:29		19	XXXXXXXXXXXXXXXXXXX
34	4:30- 4:59		18	XXXXXXXXXXXXXXXXXX
35	5:00- 5:29		18	XXXXXXXXXXXXXXXXXX
36	5:30- 5:59		18	XXXXXXXXXXXXXXXXXX
37	6:00- 6:29		17	XXXXXXXXXXXXXXXXX
38	6:30- 6:59		13	XXXXXXXXXXXXX
39	7:00- 7:29		9	XXXXXXXXX
40	7:30- 7:59		8	XXXXXXXX
41	8:00- 8:29		8	XXXXXXXX
42	8:30- 8:59		8	XXXXXXXX
43	9:00- 9:29		8	XXXXXXXX
44	9:30- 9:59		8	XXXXXXXX
45	10:00-10:29		7	XXXXXXX
46	10:30-10:59		5	XXXXX
47	11:00-11:29		5	XXXXX
48	11:30-11:59		5	XXXXX
49	12:00-12:29	a.m.	5	XXXXX
50	12:30-12:59		5	XXXXX
51	1:00- 1:29		4	XXXX
52	1:30- 1:59		4	XXXX
53	2:00- 2:29		2	XX

Figure B-2

VSP2 histogram using 30-minute periods, line 139

Period	Time		No.	
10	4:30- 4:59		4	XXXX
11	5:00- 5:29		9	XXXXXXXXX
12	5:30- 5:59		14	XXXXXXXXXXXXXX
13	6:00- 6:29		19	XXXXXXXXXXXXXXXXXXX
14	6:30- 6:59		26	XXXXXXXXXXXXXXXXXXXXXXXXXX
15	7:00- 7:29		31	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
16	7:30- 7:59		32	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
17	8:00- 8:29		30	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
18	8:30- 8:59		22	XXXXXXXXXXXXXXXXXXXXXX
19	9:00- 9:29		16	XXXXXXXXXXXXXXXX
20	9:30- 9:59		14	XXXXXXXXXXXXXX
21	10:00-10:29		13	XXXXXXXXXXXXX
22	10:30-10:59		14	XXXXXXXXXXXXXX
23	11:00-11:29		15	XXXXXXXXXXXXXXX
24	11:30-11:59		16	XXXXXXXXXXXXXXXX
25	12:00-12:29	p.m.	17	XXXXXXXXXXXXXXXXX
26	12:30-12:59		17	XXXXXXXXXXXXXXXXX
27	1:00- 1:29		17	XXXXXXXXXXXXXXXXX
28	1:30- 1:59		18	XXXXXXXXXXXXXXXXXX
29	2:00- 2:29		19	XXXXXXXXXXXXXXXXXXX
30	2:30- 2:59		23	XXXXXXXXXXXXXXXXXXXXXXX
31	3:00- 3:29		30	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
32	3:30- 3:59		31	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
33	4:00- 4:29		34	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
34	4:30- 4:59		35	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
35	5:00- 5:29		35	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
36	5:30- 5:59		35	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
37	6:00- 6:29		33	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
38	6:30- 6:59		26	XXXXXXXXXXXXXXXXXXXXXXXXXX
39	7:00- 7:29		18	XXXXXXXXXXXXXXXXXX
40	7:30- 7:59		16	XXXXXXXXXXXXXXXX
41	8:00- 8:29		14	XXXXXXXXXXXXXX
42	8:30- 8:59		13	XXXXXXXXXXXXX
43	9:00- 9:29		13	XXXXXXXXXXXXX
44	9:30- 9:59		11	XXXXXXXXXXX
45	10:00-10:29		10	XXXXXXXXXX
46	10:30-10:59		9	XXXXXXXXX
47	11:00-11:29		8	XXXXXXXX
48	11:30-11:59		8	XXXXXXXX
49	12:00-12:29	a.m.	8	XXXXXXXX
50	12:30-12:59		8	XXXXXXXX
51	1:00- 1:29		6	XXXXXX
52	1:30- 1:59		5	XXXXX
53	2:00- 2:29		2	XX

Figure B-3

Discrete VSP2 solution, line 32

End period

objective: 269.77

Start period \ End period	16	17	18	19	20	21	32	33	36	37	38	39	44	45	50	52	53
8		1.00															
9			1.00							1.00							
10					1.00						1.00		1.00				
11		1.00	1.00							1.00							
12					1.00	1.00					1.00						
13		1.00															
14	1.00							1.00									
15				1.00		1.00			1.00								
16										1.00							
23																	1.00
25										1.00							
28																1.00	
29														1.00		1.0	1.00
30							2.00				2.00			1.00	1.00		
31												1.00					

Figure B-4

Discrete VSP2 solution, line 139

Start period ↓ End period →

Objective: 438.09

	16	17	18	19	20	36	37	38	39	40	41	43	44	45	46	50	51	52	53
10			1.00				1.00				1.00				1.00				
11		2.00		1.00						1.00								1.00	
12		2.00	1.00		1.00								1.00						
13		1.00	2.00													2.00			
14	2.00		1.00	1.00				1.00				1.00					1.00		
15		3.00	1.00																1.00
16							1.00												
22																		1.00	
23										1.00									
24								1.00											
25							1.00												
28							1.00												
29						1.00													
30						1.00		2.00						1.00					
31							3.00	3.00				1.00							
32																		1.00	
33								1.00	2.00										
34																			1.00

Table B-3

Manpower Scheduling parameters
used to process the CTCUM data

Maximum number of pieces per duty	2
Maximum length of a duty (excl. travel)	8:00
Minimum length of a duty	7:00
Maximum spread for all duties	13:00
for duties finishing after midnight	11:00
for duties finishing after 2:00 a.m.	10:00
for duties beginning before 5:00 a.m.	12:00
Maximum spread for duties in which the first piece ends after 22:30	9:00
Spread premium after 10h00	:30/hour
after 12h00	1:00/hour
Overtime premium (travel only)	:30/hour
Daily time guarantee for all duties	8:00
Piece guarantee	2:00
Maximum length of any piece	6:00
No piece to be cut to begin after	23:30
Percentage of duties one-piece	25%

Table B-4

Macro file used to generate
HM2 problem matrix
for the CTCUM data

```
DEF PERIOD 0H30
DEF DAY-LENGTH 8H00
DEF MIN-WORK-TIME 7H00
DEF NUMBER-PIECES 2
DEF MAX-SPREAD 13H00
DEF MAX-SPREAD 11H30 24H00
DEF MAX-SPREAD 10H00 26H00
DEF MAX-SPREAD-START 12H00 4H30
DEF MAX-SPREAD-RELIEF 9H00 22H00
DEF SPREAD-RATE 10H00 1.5
DEF SPREAD-RATE 12H00 2.0
DEF MAX-PIECE-LENGTH 6H00
DEF END-WORKDAY-PIECE 24H00 3H30
DEF NO-PIECE-CUT 24H00 30H00
DEF GUARANTEED-PIECE 2H00
DEF MAX-FEASIBILITY 3H30
DEF BREAK-MIN-LENGTH 0H30
DEF TRIPPER-POSS NO
DEF NUMBER-RUSH-HOURS 2
DEF RUSH-HOUR 1 7H00
DEF RUSH-HOUR 2 17H00
DEF 2PIECE-PEAK 10
DEF HOURLY-RATE 1.00
DEF LUNCH-BREAK YES
CONSTRAINT 1 SPREAD 8H00 OR LESS MINIMUM 25:00
SCHEDULE LIST
(Input data)
EVALUATE-AGREEMENT
```

TABLE 8-5

Summary statistics on the
HM2 linear program

(a) Line 32

Number of rows	133
Number of columns	2,070
Number of nonzero matrix entries	44,257
Matrix density	15.96
Number of y_{qr} variables	219
Number of degrees of freedom among y_{qr}	188
Solution time (CDC Cyber/855)	134.79 sec.
Objective function at optimum	268.41
Objective function with y_{qr} variables fixed at VSP2 solution	269.77
Difference	1.36 (.50%)

(b) Line 139

Number of rows	131
Number of columns	1968
Number of nonzero matrix entries	44063
Matrix density	16.96
Number of y_{qr} variables	287
Number of degrees of freedom among y_{qr}	251
Solution time (CDC Cyber/855)	198.83 sec.
Objective function at optimum	435.57
Objective function with y_{qr} variables fixed at VSP2 solution	438.09
Difference	2.52 (.57%)

Figure B-5

Values of vehicle variables at continuous HM2 optimum, line 32

End period

objective: 268.41

Start period	16	17	18	19	20	21	32	33	36	37	38	39	44	45	50	52	53
8		.06			.94												
9						2.00											
10			.29		1.06			1.00		.65							
11		1.14					1.06									.81	
12		1.81		1.00												.19	
13	1.00																
14											1.06	.94					
15			1.71							.29			1.00				
16															1.00		
23									1.00								
25																	1.00
28																1.00	
29							.94					.06		2.00			
30										3.06	2.94						
31																	1.00

Note: An underscore indicates that variable is present in discrete VSP2 solution.

Figure B-6

Values of vehicle variables at continuous HM2 optimum, line 139

Start period ↓ End period →

Objective: 435.57

	16	17	18	19	20	36	37	38	39	40	41	43	44	45	46	50	51	52	53
10		.33	1.75				1.33										.58		
11							5.00												
12	1.79	2.83					.38												
13	.21	3.52							1.27										
14		1.32	1.16	2.00	1.00			1.38								.14			
15			3.08					1.92											
16																1.00			
22						.86			.14										
23														1.00					
24						1.00													
25															1.00				
28											1.00								
29												1.00							
30								4.00											
31						.14						1.00				.86		3.00	2.00
32							.29	.71										1.00	
33										2.00			1.00						
34									.58								.42		

Note: An underscore indicates that variable is present in discrete VSP2 solution.

Figure B-7

Reduced costs associated with vehicle variables
at continuous HM2 optimum, line 32

End period — objective: 268.41

Start period	16	17	18	19	20	21	32	33	36	37	38	39	44	45	50	52	53
8	*	b	*	*	b	n	*	*	*	*	*	*	*	*	*	*	*
9	n	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
10	*	*	b	*	b	*	*	b	*	b	*	*	*	*	*	*	*
11	*	b	*	*	*	*	b	*	*	*	*	*	*	*	*	b	*
12	n	b	*	b	*	*	*	*	*	*	*	*	*	*	*	b	*
13	b	*	*	*	*	*	*	*	*	*	*	*	*	*	n	*	*
14	.36	*	*	*	*	*	*	*	*	*	b	b	*	*	*	*	*
15	.73	.36	b	*	*	*	*	*	*	b	*	*	b	*	*	*	*
16	1.23	.73	*	n	*	*	*	*	*	*	*	*	*	*	b	*	*
23							*	n	*	*	*	*	*	*	*	*	*
25							*	*	*	*	*	*	*	*	*	n	*
28							*	n	n	*	*	*	*	*	*	*	*
29							b	*	*	*	*	b	*	b	*	*	*
30							.50	*	*	b	b	*	*	*	*	*	*
31							.96	.50	*	n	*	*	*	*	*	*	b

Note : All reduced costs except those with numeric values in table are zero. 'b' indicates variable which is basic at optimum; 'n' indicates a variable which is basic with zero value at optimum. An underscore indicates that variable is present in discrete VSP" solution.

Figure B-8

Reduced costs associated with vehicle variables
at continuous HM2 optimum, line 139

Start period ↓ End period →

Objective: 435.57

	16	17	18	19	20	36	37	38	39	40	41	43	44	45	46	50	51	52	53
10	.17	b	b	*	*	*	b	*	*	*	*	*	*	*	*	*	b	*	*
11	.17	*	*	*	*	n	b	*	*	*	*	*	*	*	*	*	*	*	*
12	b	b	*	*	*	*	b	*	*	*	*	*	*	*	*	*	*	*	*
13	b	b	*	*	*	*	*	*	b	*	*	*	*	*	*	*	*	*	*
14	.39	b	b	b	b	*	*	b	*	*	*	*	*	*	*	b	*	*	*
15	.61	n	b	*	*	*	*	b	*	*	*	*	*	*	*	*	*	*	*
16	n	n	.22	*	*	*	*	*	*	*	n	*	*	*	*	b	*	*	*
22						b	*	*	b	*	*	*	*	*	*	*	*	*	*
23						*	n	*	*	*	*	*	*	b	*	*	*	*	*
24						b	*	*	*	*	*	*	*	*	*	*	n	*	*
25						*	n	*	*	*	n	*	*	*	b	*	*	*	*
28						*	*	*	*	*	b	*	*	*	*	*	*	*	*
29						*	*	*	*	*	*	b	*	*	n	*	n	*	*
30						*	*	b	*	*	*	*	*	n	*	*	n	*	*
31						b	*	*	*	*	*	b	*	*	*	b	*	b	b
32						*	b	b	*	*	*	*	*	*	*	*	*	*	*
33						.08	.67	n	n	b	*	*	b	*	*	*	*	n	*
34						*	*	.17	b	*	*	*	*	*	*	*	b	*	*

Note: All reduced costs except those with numeric values in table are zero. 'b' indicates variable which is basic at optimum; 'n' indicates a variable which is basic with zero value at optimum. An underscore indicates that variable is present in discrete VSP2 solution.

Figure B-9

Optimal vehicle variable set "closest" to VSP2 solution, line 32

End period

objective: 268.41

Start period	16	17	18	19	20	21	32	33	36	37	38	39	44	45	50	52	53
8		1.00															
9			1.00							1.00							
10	1.00				R		1.00				R		1.00				
11		1.00	1.00							1.00							
12					1.00	1.00					1.00						
13		1.00															
14	R				1.00		1.00	R									
15				1.00		1.00			1.00								
16										1.00							
23																	1.00
25										1.00							
28																1.00	
29														1.00		1.00	1.00
30							R	1.00			3.00			1.00	1.00		
31												1.00					

Note: An underscore indicates that variable is present in discrete VSP2 solution. 'R' indicates a variable "rejected" by HM2 model.

Figure B-10

Optimal vehicle variable set "closest" to VSP2 solution, line 32

Start period ↓ End period →

Objective: 435.57

	16	17	18	19	20	36	37	38	39	40	41	43	44	45	46	50	51	52	53
10			1.00				1.00				1.00				1.00				
11		2.00		1.00						1.00								1.00	
12	.52	2.66	.82		R								1.00						
13	1.48	1.00	R		.52											2.00			
14	R	2.34	.18	1.00	.48			1.00				1.00					1.00		
15		R	4.00																1.00
16							1.00												
22																		1.00	
23									1.00	R									
24						1.00		R											
25							1.00												
28							1.00												
29						1.00													
30						R		4.00						R					
31							3.00	3.00				1.00							
32																		1.00	
33								R	1.00	1.00				1.00					
34																			1.00

Note: An underscore indicates that variable is present in discrete VSP2 solution. 'R' indicates a variable "rejected" by HM2 model.

Table B-6

Summary of improvement heuristic, line 32

Step	Type	Details	Minutes added	Effect on objective
1	Basis change	Move to solution of Figure B-9	-	-
2	Rehooking	SU(41) = 60 SU(43) = 0 PR(50) = 41 [14-16] [10-20] → [10-16] [14-20]	4	-
3	Rehooking	SU(162) = 183 SU(163) = 0 PR(183) = 162 [30-32] [10-38] → [10-32] [30-38]	3	-
4	Rehooking	PR(169) = 149 PR(167) = 0 SU(149) = 169 [30-32] [23-53] → [23-32] [30-53]	4	-

Total minutes added	11 (.07%)
Total hours added (11/60)	.18
Objective function increase	-
Total cost increase over optimum (.0 + .18)	.18
Percentage recuperated 100 x (1.0 - (.18/1.36))	86.8%

Table B-7

Summary of improvement heuristic, line 139

Step	Type	Details	Minutes added	Effect on objective
1	Basis change	Move to solution of Figure B-10	-	-
2	Rehooking	SU(59) = 88 SU(61) = 0 PR(88) = 59 [15-17] [12-17] [12-20] → [15-20]	3	-
3	Rehooking	SU(64) = 90 SU(65) = 0 PR(90) = 64 [15-17] [10-17] [10-41] → [15-41]	2	-
4	Rehooking	SU(68) = 93 SU(71) = 0 PR(93) = 68 [15-17] [12-18] [12-44] → [15-44]	5	-
5	Basis change	Fix [24-38] at 1.0	-	-
6	Basis change	Fix [15-18] at 1.0	-	-
7	Rehooking	SU(351) = 368 SU(352) = 0 PR(368) = 351 [33-39] [33-45] [30-45] → [30-39]	4	-
8	Basis change	Fix [12-18] at 1.0	-	-
9	Rehooking	SU(347) = 366 SU(349) = 0 PR(366) = 347 [33-38] [33-52] [22-52] → [22-38]	4	-
10	Rehooking	PR(230) = 210 PR(228) = 0 SU(210) = 230 [30-36] [30-53] [15-53] → [15-36]	4	-
11	Basis change	Fix [30-38] at 2.0	-	-
12	Basis change	Fix [14-16] at 2.0	-	+ .82

Total minutes added	22 (.09$)
Total hours added (22/60)	.37
Objective function increase	.82
Total cost increase over optimum (.37 + .82)	1.19
Percentage recuperated 100 x (1.0 - (1.19/2.52))	53.8$

Figure 8-11

Vehicle variable values after application of improvement heuristic, line 32

End period

objective: 268.41

Start period	16	17	18	19	20	21	32	33	36	37	38	39	44	45	50	52	53
8		1.00															
9			1.00							1.00							
10		1.00					1.00						1.00				
11		1.00	1.00							1.00							
12					1.00	1.00					1.00						
13		1.00															
14					1.00			1.00									
15				1.00		1.00			1.00								
16										1.00							
23							1.00										
25										1.00							
28																1.00	
29														1.00		1.00	1.00
30											3.00			1.00	1.00		1.00
31												1.00					

Note: An underscore indicates that variable is present in discrete VSP2 solution.

Figure B-12

Vehicle variable values variables after application of improvement heuristic, line 139

Start period ↓ End period →

Objective: 436.39

	16	17	18	19	20	36	37	38	39	40	41	43	44	45	46	50	51	52	53
10		1.00	1.00				1.00				1.00				1.00				
11		2.00		1.00						1.00								1.00	
12		4.00	1.00																
13		1.00	1.00													2.00			
14	2.00		1.00	1.00				1.00				1.00					1.00		
15			1.00		1.00	1.00					1.00		1.00						
16							1.00												
22								1.00											
23										1.00									
24								1.00											
25							1.00												
28							1.00												
29						1.00													
30								2.00	1.00										1.00
31							3.00	3.00				1.00							
32																		1.00	
33									1.00					1.00				1.00	
34																			1.00

Note: An underscore indicates that variable is present in discrete VSP2 solution.

Table B-8

Improved vehicle solution summary, line 32

Block	Garage(Per)	Start Time/Loc	End Time/Loc	Garage(Per)	Vehicle time
1	3:58 (8)	4:25/210	8:20/210	8:46 (17)	4:48
2	4:13 (9)	4:40/210	8:50/210	9:16 (18)	5:03
3	4:27 (9)	4:54/210	18:35/209	18:58 (37)	14:31
4	4:41 (10)	5:08/210	15:51/209	16:14 (32)	11:33
5	4:49 (10)	5:16/210	21:41/209	22:04 (44)	17:15
6	4:56 (10)	5:23/210	8:04/210	8:30 (17)	3:34
7	5:04 (11)	5:31/210	8:12/210	8:38 (17)	3:34
8	5:16 (11)	5:43/210	8:36/210	9:02 (18)	3:46
9	5:22 (11)	5:49/210	18:20/209	18:43 (37)	13:21
10	5:35 (12)	6:02/210	9:37/209	10:00 (20)	4:25
11	5:47 (12)	6:14/210	18:43/209	19:06 (38)	13:19
12	5:59 (12)	6:26/210	10:07/209	10:30 (21)	4:31
13	6:29 (13)	6:56/210	8:28/210	8:54 (17)	2:25
14	6:50 (14)	7:15/209	9:35/210	10:01 (20)	3:11
15	6:58 (14)	7:23/209	16:18/209	16:41 (33)	9:43
16	7:06 (15)	7:31/209	10:29/209	10:52 (21)	3:46
17	7:14 (15)	7:39/209	9:07/209	9:30 (19)	2:16
18	7:22 (15)	7:47/209	17:59/209	18:22 (36)	11:00
19	7:30 (16)	7:55/209	18:10/209	18:33 (37)	11:03
20	11:12 (23)	11:37/209	16:03/209	16:26 (32)	5:14
21	12:03 (25)	12:28/209	18:30/209	18:53 (37)	6:50
22	13:51 (28)	14:18/210	25:54/209	26:17 (52)	12:26
23	14:08 (29)	14:35/210	26:11/209	26:34 (53)	12:26
24	14:23 (29)	14:50/210	22:28/209	22:51 (45)	8:28
25	14:25 (29)	14:50/209	25:37/209	26:00 (52)	11:35
26	14:31 (30)	14:58/210	25:05/209	25:28 (50)	10:57
27	14:35 (30)	15:02/210	18:51/209	19:14 (38)	4:39
28	14:42 (30)	15:09/210	18:55/209	19:18 (38)	4:36
29	14:46 (30)	15:11/209	22:09/209	22:32 (45)	7:46
30	14:46 (30)	15:13/210	26:28/209	26:51 (53)	12:05
31	14:53 (30)	15:20/210	19:03/209	19:26 (38)	4:33
32	15:01 (31)	15:28/210	19:13/209	19:36 (39)	4:35
				TOTAL VEHICLE TIME:	249:14

Note: The (per) column indicates discrete time period.

Table B-9

Improved vehicle solution summary, line 139

Block	Garage(Per)	Start Time/Loc	End Time/Loc	Garage(Per)	Vehicle time
1	4:30 (10)	4:55/739	8:36/740	9:06 (18)	4:36
2	4:38 (10)	5:10/740	22:40/739	23:05 (46)	18:27
3	4:45 (10)	5:10/739	18:28/740	18:58 (37)	14:13
4	5:00 (11)	5:25/739	26:03/739	26:28 (52)	21:28
5	4:58 (10)	5:30/740	8:24/740	8:54 (17)	3:56
6	5:08 (11)	5:33/739	9:08/740	9:38 (19)	4:30
7	5:13 (11)	5:38/739	8:30/739	8:55 (17)	3:42
8	5:19 (11)	5:44/739	19:46/739	20:11 (40)	14:52
9	5:28 (11)	5:53/739	8:03/740	8:33 (17)	3:05
10	5:32 (12)	5:57/739	8:08/740	8:38 (17)	3:06
11	5:36 (12)	6:01/739	8:13/740	8:43 (17)	3:07
12	5:41 (12)	6:06/739	8:19/740	8:49 (17)	3:08
13	5:49 (12)	6:14/739	8:30/740	9:00 (18)	3:11
14	5:53 (12)	6:18/739	8:32/740	9:02 (18)	3:09
15	6:02 (13)	6:27/739	8:42/740	9:12 (18)	3:10
16	6:07 (13)	6:32/739	24:35/739	25:00 (50)	18:53
17	6:17 (13)	6:42/739	8:57/740	9:27 (18)	3:10
18	6:23 (13)	6:48/739	25:04/739	25:29 (50)	19:06
19	6:26 (13)	6:51/739	8:22/739	8:47 (17)	2:21
20	6:32 (14)	6:57/739	18:46/740	19:16 (38)	12:44
21	6:37 (14)	7:)2/739	9:15/740	9:45 (19)	3:08
22	6:39 (14)	7:04/739	21:10/739	21:35 (43)	14:56
23	6:41 (14)	7:06/739	7:51/740	8:21 (16)	1:40
24	6:46 (14)	7:11/739	7:57/740	8:27 (16)	1:41
25	6:54 (14)	7:19/739	25:21/739	25:46 (51)	18:52
26	6:47 (14)	7:19/740	8:49/740	9:19 (18)	2:32
27	7:00 (15)	7:25/739	17:45/740	18:15 (36)	11:15
28	7:05 (15)	7:30/739	9:48/740	10:18 (20)	3:13
29	7:11 (15)	7:36/739	20:19/739	20:44 (41)	13:33
30	7:16 (15)	7:41/739	21:51/739	22:16 (44)	15:00
31	7:25 (15)	7:50/739	8:34/740	9:04 (18)	1:39
32	7:31 (16)	7:56/739	18:08/740	18:38 (37)	11:07
33	10:39 (22)	11:04/730	19:07/739	19:32 (39)	8:53
34	11:06 (23)	11:31/739	19:37/739	20:02 (40)	8:56
35	11:40 (24)	12:05/739	18:42/739	19:07 (38)	7:27
36	12:14 (25)	12:39/739	18:21/740	18:51 (37)	6:37
37	13:50 (28)	14:22/740	18:29/739	18:54 (37)	5:04
38	14:07 (29)	14:39/740	17:58/740	18:28 (36)	4:21
39	14:35 (30)	15:07/740	19:16/739	19:41 (39)	5:06
40	14:51 (30)	15:16/739	26:30/740	26:55 (53)	12:04
41	14:45 (30)	15:17/740	18:36/740	19:06 (38)	4:21
42	14:51 (30)	15:23/740	18:41/740	19:11 (38)	4:20
43	15:09 (31)	15:34/739	18:50/739	19:15 (38)	4:06
44	15:01 (31)	15:33/740	18:51/740	19:21 (38)	4:20
45	15:16 (31)	15:41/739	18:06/740	18:36 (37)	3:20
46	15:08 (31)	15:40/740	18:55/740	19:25 (38)	4:17
47	15:26 (31)	15:51/739	18:15/740	18:45 (37)	3:19
48	15:18 (31)	15:50/740	18:18/739	18:43 (37)	3:25
49	15:25 (31)	15:57/740	21:31/739	21:56 (43)	6:31
50	15:38 (32)	16:10/740	25:40/739	26:05 (52)	10:27
51	16:04 (33)	16:36/740	25:52/739	26:17 (52)	10:13
52	16:15 (33)	16:47/740	22:10/739	22:35 (45)	5:20
53	16:25 (33)	16:57/740	19:24/739	19:49 (39)	3:24
54	16:35 (34)	17:07/740	26:15/739	26:40 (53)	10:05
				TOTAL VEHICLE TIME:	399:26

Note : The (per) column indicates discrete time period.

Table B-10

Relief point information for the two CTCUM lines

(a) Line 32

Running time interval (minutes)	Minutes from 209 to relief	Minutes from 210 to relief
30-31	-	17
32-33	14	20
34	14	21
35	15	21
36	15	22
37	16	22
38-39	17	22
40-42	18	24
43	19	24
44	20	25
45-46	20	26
47	21	27

Minutes from relief to Anjou garage: 36 allowed
13 paid

Minutes from relief to itself: 10 paid and allowed

(b) Line 139

Running time interval (minutes)	Minutes from 739 to relief	Minutes from 740 to relief
27-30	12	17
31-32	13	17
33	14	18
34	14	19
35	14	20
36-38	16	20
39	16	21
40	16	22
41	17	22
42	17	23
43	18	23
44-47	-	25
48-49	-	29

Minutes from relief to Anjou garage: 47 allowed
23 paid

Minutes from relief to itself: 10 paid and allowed

Table B-11

Parameters used to create manpower assignments for the CTCUM data

(a) Macro file

```
DEF PERIOD 0h34
DEF DAY-LENGTH 7h56
DEF MIN-WORK-TIME 6h48
DEF NUMBER-PIECES 2
DEF MAX-SPREAD 13h02
DEF MAX-SPREAD 11h20 23h48
DEF MAX-SPREAD 9h38 26h04
DEF MAX-SPREAD-START 11h54 4h32
DEF MAX-SPREAD-RELIEF 9h04 22h06
DEF SPREAD-RATE 9h38 1.5
DEF SPREAD-RATE 11h54 2.0
DEF MAX-PIECE-LENGTH 6h14
DEF END-WORKDAY-PIECE 23h48 3h24
DEF NO-PIECE-CUT 23h48 29h28
DEF GUARANTEED-PIECE 2h16
DEF MAX-FEASIBILITY 5h40
DEF BREAK-MIN-LENGTH 0h34
DEF TRIPPER-POSS NO
DEF NUMBER-RUSH-HOURS 2
DEF RUSH-HOUR 1 7h22
DEF RUSH-HOUR 2 16h26
DEF 2PIECE-PEAK 10
DEF HOURLY-RATE 10.51
DEF FRINGE BENEFITS 50
DEF LUNCH-BREAK YES
SCHEDULE LIST
(INPUT DATA)
EVALUATE-AGREEMENT
```

(b) Selection file

Description	Attribute	Penalty
1	Tripper Platform time between 5:00 and 8:00	9999.99
2	Tripper Begins after 10:00 p.m.	9999.99
3	Tripper Ends before 5:00 a.m.	9999.99
4	3-piece workday	9999.99

Table B-12

Summary of manpower scheduling using CTCUM data

(a) Line 32

	VSP2 vehicle solution		Adjusted vehicle solution	
	No.	Avg. platform	No.	Avg. platform
One-piece duties	9	7:50	8	7:49
Two-piece duties	23	7:40	24	7:37
Trippers	1	2:16	1	3:46
Cost of regular duties		$3303.36		$3287.04
Cost of trippers		35.79		59.28
Total cost		$3339.15		$3346.32

(b) Line 139

	VSP2 vehicle solution		Adjusted vehicle solution	
	No.	Avg. platform	No.	Avg. platform
One-piece duties	13	7:42	13	7:42
Two-piece duties	37	7:42	37	7:38
Trippers	7	13:51	8	19:05
Cost of regular duties		$5265.50		$5171.50
Cost of trippers		218.35		300.80
Total cost		$5483.85		$5472.30

Table B-13

Same estimates of potential for improving
a vehicle solution using HM2 model

	"basic" constraints	Max spread 11:30	% Δ	Premium 2.0 after 11:00	% Δ	Pce guar 3:00	% Δ	% Straight 35%	% Δ
Line 32									
optimal	268.41	270.76	.9%	269.29	.56%	268.41	-	268.41	-
fixed at VSP2	269.77	271.34	.58%	271.24	.54%	272.30	.94%	269.77	-
Difference	1.36(.50%)	.58(.21%)		1.32(.49%)		3.89(1.43%)		1.36(.50%)	
Line 139									
optimal	435.57	495.84	13.84%	444.23	1.99%	438.20	.60%	435.95	.09%
fixed at VSP2	438.09	496.99	13.44%	445.91	1.79%	444.59	1.48%	439.50	.32%
Difference	2.52(.57%)	1.15(.23%)		1.68(.38%)		6.39(1.44%)		3.55(.81%)	

Note: % Δ columns refer to percentage difference from "basic" constraints column.

Figure B-13

VSP2 histograms using one-hour periods

(a) Line 32

Period	Time		No.	
4	3:00- 3:59	a.m.	1	X
5	4:00- 4:59		6	XXXXXX
6	5:00- 5:59		12	XXXXXXXXXXXX
7	6:00- 6:59		15	XXXXXXXXXXXXXXX
8	7:00- 7:59		19	XXXXXXXXXXXXXXXXXXX
9	8:00- 8:59		15	XXXXXXXXXXXXXXX
10	9:00- 9:59		12	XXXXXXXXXXXX
11	10:00-10:59		8	XXXXXXXX
12	11:00-11:59		9	XXXXXXXXX
13	12:00-12:59	p.m.	10	XXXXXXXXXX
14	1:00- 1:59		11	XXXXXXXXXXX
15	2:00- 2:59		20	XXXXXXXXXXXXXXXXXXXX
16	3:00- 3:59		21	XXXXXXXXXXXXXXXXXXXXX
17	4:00- 4:59		18	XXXXXXXXXXXXXXXXXX
18	5:00- 5:59		18	XXXXXXXXXXXXXXXXXX
19	6:00- 6:59		13	XXXXXXXXXXXXX
20	7:00- 7:59		8	XXXXXXXX
21	8:00- 8:59		8	XXXXXXXX
22	9:00- 9:59		8	XXXXXXXX
23	10:00-10:59		5	XXXXX
24	11:00-11:59		5	XXXXX
25	12:00-12:59	a.m.	5	XXXXX
26	1:00- 1:59		4	XXXX

K = { 4,5,6,7,8,12,13,14,15,16 }

L = { 8,9,10,16,18,19,22,25,26 }

(b) Line 139

Period	Time		No.	
5	4:00- 4:59	a.m.	4	XXXX
6	5:00- 5:59		14	XXXXXXXXXXXXXX
7	6:00- 6:59		26	XXXXXXXXXXXXXXXXXXXXXXXXXX
8	7:00- 7:59		32	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
9	8:00- 8:59		22	XXXXXXXXXXXXXXXXXXXXXX
10	9:00- 9:59		14	XXXXXXXXXXXXXX
11	10:00-10:59		14	XXXXXXXXXXXXXX
12	11:00-11:59		16	XXXXXXXXXXXXXXXX
13	12:00-12:59	p.m.	17	XXXXXXXXXXXXXXXXX
14	1:00- 1:59		18	XXXXXXXXXXXXXXXXXX
15	2:00- 2:59		23	XXXXXXXXXXXXXXXXXXXXXXX
16	3:00- 3:59		31	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
17	4:00- 4:59		35	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
18	5:00- 5:59		35	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
19	6:00- 6:59		26	XXXXXXXXXXXXXXXXXXXXXXXXXX
20	7:00- 7:59		16	XXXXXXXXXXXXXXXX
21	8:00- 8:59		13	XXXXXXXXXXXXX
22	9:00- 9:59		11	XXXXXXXXXXX
23	10:00-10:59		9	XXXXXXXXX
24	11:00-11:59		8	XXXXXXXX
25	12:00-12:59	a.m.	8	XXXXXXXX
26	1:00- 1:59		5	XXXXX

K = { 5,6,7,8,11,12,13,14,15,16,17 }

L = { 8,9,10,18,19,20,21,22,23,25,26 }

Table B-14

Macro file using one-hour periods

```
DEF PERIOD 1H00
DEF DAY-LENGTH 8H00
DEF NUMBER-PIECES 2
DEF HOURLY-RATE 1.00
DEF MAX-SPREAD-RELIEF 9H00 22H00
DEF END-WORKDAY-PIECE 24H00 4H00
DEF MAX-SPREAD-START 12H00 5H00
DEF GUARANTEED-PIECE 2H00
DEF SPREAD-RATE 10H00 1.5
DEF SPREAD-RATE 12H00 2.0
DEF MAX-SPREAD 11H00
DEF MAX-SPREAD 11H00 24H00
DEF MAX-SPREAD 10H00 26H00
DEF NUMBER-RUSH-HOURS 2
DEF RUSH-HOUR 1 7H00
DEF RUSH-HOUR 2 17H00
DEF 2PIECE-PEAK 10
DEF BREAK-MIN-LENGTH 1H00
DEF MAX-FEASIBILITY 3H00
DEF LUNCH-BREAK YES
DEF MAX-PIECE-LENGTH 6H00
DEF MIN-WORK-TIME 7H00
DEF NO-PIECE-CUT 24H00 30H00
DEF LP-TECHNIC 1
CONSTRAINT 1 SPREAD 8H00 OR LESS MINIMUM 25.00
SCHEDULE LIST
(INPUT DATA)
END
CHARAC-SCHEDULE
EVALUATE-AGREEMENT
```

Table B-15

Summary statistics for dual variable estimation problem

(a) Primal linear program

	Line 32	Line 139
Number of rows (including objective)	60	59
Number of columns	288	295
Number of nonzero matrix elements	3779	4159
Matrix density	21.79	23.82
Number of simplex pivots		
Phase I	37	35
Phase II	78	80
Objective function at optimum	268.61	433.67
Solution time (including input and output) on CDC Cyber/855	21.37 sec.	22.98 sec.

(b) Quadratic program

	Line 32	Line 139
Number of rows (including objective)	290	297
Number of columns	59	58
Number of nonzero matrix elements	3614	4010
Matrix density	20.77	22.88
Number of iterations	28	34
Objective function at optimum	47.61	45.03
Solution time (including input and output) on CDC Cyber/855	23.47 sec.	26.91 sec.

Table B-16

Dual variables from HM2, line 32

	Before adjustment	After adjustment
Block-covering constraints		
Period 4	.72	.72
5	1.22	1.22
6	1.22	1.22
7	1.78	1.78
8	1.22	1.22
9	.72	.72
10	.72	.72
11	.72	.72
12	.72	.72
13	.72	.72
14	.72	.72
15	1.22	1.22
16	1.22	1.22
17	1.22	1.22
18	1.78	1.78
19	.72	.72
20	.72	.72
21	.72	.72
22	.72	.72
23	-	.72
24	-	.72
25	2.67	1.22
26	1.22	1.22
Percentage straight duties	-	-
Small-block constraints		
Periods 6 to 8	-	-
7 to 8	-	-
7 to 9	-	-
8 to 8	-	-
8 to 9	-	-
8 to 10	-	-
14 to 16	-	-
15 to 16	-	-
16 to 16	-	-
16 to 18	-	-
Service-covering constraints		
Period 4	.72	.72
5	1.22	1.22
6	1.22	1.22
7	1.78	1.78
8	1.22	1.22
9	.72	.72
10	.72	.72
11	.72	.72
12	.72	.72
13	.72	.72
14	.72	.72
15	1.22	1.22
16	1.22	1.22
17	3.00	1.50
18	-	1.50
19	.72	.72
20	2.17	.72
21	-	.72
22	-	.72
23	2.67	.89
24	-	.89
25	-	.89
26	1.22	1.22

Table B-17

Dual variables from HM2, line 139

	Before adjustment	After adjustment
Block-covering constraints		
Period 5	.67	.67
6	1.50	1.50
7	1.50	1.50
8	1.50	1.50
9	1.17	1.17
10	.67	.67
11	.67	.67
12	.67	.67
13	.67	.67
14	.67	.67
15	.67	.67
16	1.17	1.17
17	1.50	1.50
18	1.50	1.50
19	1.50	1.50
20	.67	.67
21	.67	.67
22	.67	.67
23	.67	.67
24	-	.67
25	1.33	.67
26	1.17	1.17
Percentage straight duties	-	-
Small-block constraints		
Periods 6 to 8	-	-
7 to 8	-	-
7 to 9	-	-
8 to 8	-	-
8 to 9	-	-
8 to 10	-	-
16 to 18	-	-
17 to 18		
17 to 19		
Pull-out constraint		
Period 11	-	-
Service-covering constraints		
Period 5	.67	.67
6	1.50	1.50
7	1.50	1.50
8	1.50	1.50
9	1.17	1.17
10	.67	.67
11	.67	.67
12	.67	.67
13	.67	.67
14	.67	.67
15	.67	.67
16	1.17	1.17
17	3.00	1.50
18	-	1.50
19	1.50	1.50
20	.67	.67
21	.67	.67
22	.67	.67
23	.67	.67
24	1.33	.67
25	-	.67
26	1.17	1.17

Appendix C

A FORTRAN program to solve VSP2

```
      PROGRAM VSP2(INPUT,TAPE5=INPUT,OUTPUT,TAPE6=OUTPUT,
     *             TAPE7=65,TAPE8=65)
C
C     THIS PROGRAM ACCEPTS TRIP INFORMATION AND SOLVES THE VSP2
C     TRANSHIPMENT PROBLEM USING RNET (GRIGORIADIS AND HSU, 1979).
C     THIS VERSION IS FOR THE CDC FTN5 COMPILER.
C
C     INPUT (UNIT 7):
C        FIRST RECORD:
C              ANODE : NUMBER OF FIRST TERMINUS
C              SORA  : MINUTES FROM GARAGE TO FIRST TERMINUS
C              RETA  : MINUTES FROM FIRST TERMINUS TO GARAGE
C        SECOND RECORD:
C              BNODE : NUMBER OF SECOND TERMINUS
C              SORB  : MINUTES FROM GARAGE TO SECOND TERMINUS
C              RETB  : MINUTES FROM SECOND TERMINUS TO GARAGE
C        THIRD RECORD:
C              PERLEN: PERIOD LENGTH IN MINUTES (FOR REPORT)
C              SHIFT : MINUTES FROM MIDNIGHT TO START OF FIRST
C                      PERIOD (FOR REPORT)
C              REP   : GARAGE PREPARATION TIME (MINUTES)
C              CODE  : 0, INPUT TIMES ARE IN MINUTES
C                      1, INPUT TIMES ARE IN HOURS AND MINUTES
C        SUBSEQUENT RECORDS:
C             DEPHR(II) : START TIME FOR TRIP II
C             FINHR(II) : END TIME FOR TRIP II
C             DEPEN(II) : START LOCATION FOR TRIP II
C             FINEN(II) : END LOCATION FOR TRIP II
C             LAST RECORD MUST HAVE '9999' IN COLS 1-4
C     NOTE:  ALL INPUT FIELDS ARE (I4) AND
C            ARE CONTIGUOUS WITHIN RECORDS
C     OUTPUT:
C        UNIT 6 (PRINTER):
C              TRIP SOLUTION CONSISTING OF, FOR EACH TRIP
C                 II:
C                - PREDECESSOR TRIP
C                - START LOCATION
C                - START TIME
C                - END TIME
C                - END LOCATION
C                - SUCCESSOR TRIP
C                - PERIOD IN WHICH BLOCK STARTS
C                - PERIOD IN WHICH BLOCK WOULD END IF TRIP
C                  II WERE LAST TRIP
C        UNIT 8:
C             SAME AS INPUT EXCEPT WITH SUCCESSOR TACKED ON EACH
C             TRIP RECORD AS FIFTH (I4) FIELD
C     LIMITATIONS:  MAX TRIPS 500
C                   MAX ARCS 10000
C                   MAX NODES 1500
C
      IMPLICIT INTEGER(A-Z)
      REAL RCOUT,QUAD,TMULT
      REAL FRQ,PO,PBAR,CP,RTT,TEO
      LOGICAL SORTIE
      DIMENSION P(1500),F(1500),D(1500),U(1500),ARC(1500)
      DIMENSION X(1500),R(1500),PCAND(1)
      DIMENSION FROM(10000),TO(10000),H(10000),C(10000)
      DIMENSION RTN(20),LPR(12),RTT(2)
```

```
      DIMENSION DEBHR(500),FINHR(500),DEBEN(500),FINEN(500)
      DIMENSION PRCHII(500),DERNII(500)
      DIMENSION B(500)
      DIMENSION IREVFL(4)
      COMMON /A/FROM /B/TO /C/C /D/H /E/P
      COMMON /F/F /G/D /H/U /I/ARC /J/X
      COMMON /K/R /N/PCAND
      COMMON /L/BT,MW,PT,M,N,IRT,MXIT,FRQ,PO,PBAR,CP,PAS,RTN,
     .  LPR,RTT,NCAND
      COMMON /M/IREVFL,KARC,ICOM,ITH,MAXF,LNOD,MIC,MAC,NPE,
     .         IT,ITO,LGAM,ISDB,TEO
      COMMON //DEBHR,FINHR,DEBEN,FINEN,PRCHII,DERNII
      BT = 48
      IRT = 1
      PT = 0
      MXIT = 20000
      FRQ = 2.1
      PO = 1.0
      PBAR = 1.5
      NCAND = 0
      AMENDE = 10000
      GRONUM = 9999
      QUAD = 0.01
      TMULT = 10.0
      MAXARC = 10000
      MAXNOD = 1500
*
*     READ IN DATA
*
      READ(7,10) ANODE,SORA,RENA
      READ(7,10) BNODE,SORB,RENB
      READ(7,10) PERLEN,SHIFT,REP,CODE
   10 FORMAT(5I4)
*
      II = 0
   50 CONTINUE
      II = II + 1
      READ(7,55) DEBHR(II),FINHR(II),DEBEN(II),FINEN(II)
   55 FORMAT(4I4)
      IF (DEBHR(II).EQ.GRONUM) GO TO 60
      IF (CODE.EQ.1) DEBHR(II) = CONV(DEBHR(II))
      IF (CODE.EQ.1) FINHR(II) = CONV(FINHR(II))
      GO TO 50
*
   60 CONTINUE
      NVOY = II -1
      NARC = 0
*
*     SET UP TRIP NODES AND TAKE INFORMATION FOR GARAGE NODE
*     COMPRESSION
*
      DO 160 II = 1,NVOY
      PRCHII(II) = 0
      DERNII(II) = 0
      FIN = FINHR(II)
      FINII = 2*II
      DEB = DEBHR(II)
      DEBII = FINII - 1
      X(DEBII) = -1
      X(FINII) = 1
      GARSOR = SORA
```

```
      IF (DEBEN(II).EQ.BNODE) GARSOR = SORB
      GARREN = RENA
      IF (FINEN(II).EQ.BNODE) GARREN = RENB
      D(DEBII) = DEB - GARSOR
      U(DEBII) = II
      D(FINII) = -(FIN + GARREN)
      U(FINII) = -II
*     WRITE(6,99) DEBII,U(DEBII),D(DEBII),FINII,U(FINII),D(FINII)
*  99 FORMAT(' DEBII = ',I3,' U = ',I5,' D = ',I5,
*    .       ' FINII = ',I3,' U = ',I5,' D = ',I5)
*
*   SET UP TRIP HOOK ARCS
*
      I1 = II + 1
      DO 150 JJ = I1,NVOY
      DEBJJ = 2*JJ - 1
      GARSOR = SORA
      IF (DEBEN(JJ).EQ.BNODE) GARSOR = SORB
      LAY = DEBHR(JJ) - FIN
      IF (LAY.LT.0) GO TO 150
      IF (LAY.GT.(GARREN + GARSOR + 2*REP)) GO TO 150
      IF (DEBEN(JJ).NE.FINEN(II)) GO TO 150
      NARC = NARC + 1
      IF (NARC.GT.MAXARC) GO TO 900
      FROM(NARC) = FINII
      TO(NARC) = DEBJJ
      RCOUT = FLOAT(LAY)
      RCOUT = RCOUT*(1. + RCOUT*QUAD) * TMULT
      C(NARC) = INT(RCOUT)
      H(NARC) = 1
  150 CONTINUE
*
  160 CONTINUE
*
*   SORT GARAGE TIMES
*
      K = 2*NVOY
      DO 170 I = 1,K
      ITEMPS = IABS(D(I))
      I1 = I + 1
      DO 165 J = I1,K
      JTEMPS = IABS(D(J))
      IF (JTEMPS.GE.ITEMPS) GO TO 165
      TTEMPS = D(I)
      TII = U(I)
      D(I) = D(J)
      U(I) = U(J)
      D(J) = TTEMPS
      U(J) = TII
      ITEMPS = IABS(D(I))
  165 CONTINUE
  170 CONTINUE
      DO 178 I = 1,K
*     WRITE (6,177) I,U(I),D(I)
* 177 FORMAT(' I = ',I3,' U = ',I5,' D = ',I5)
  178 CONTINUE
*
*   SET UP GARAGE NODES, PULL-OUT, PULL-IN ARCS
*
      I = 0
      MULT = INT(TMULT)
```

```
      KK = K + 1
      X(KK) = 0
      SORTIE = .FALSE.
  180 CONTINUE
      I = I + 1
      IF (I.GT.K) GO TO 200
      TII = IABS(U(I))
      FINII = 2*TII
      DEBII = FINII - 1
      IF (D(I)) 185,190,190
  185 CONTINUE
      IF (.NOT.SORTIE) GO TO 187
      NARC = NARC + 1
      IF (NARC.GT.MAXARC) GO TO 900
      FROM(NARC) = KK
      TO(NARC) = KK + 1
      C(NARC) = 0
      H(NARC) = NVOY
      KK = KK + 1
      IF (KK.GT.MAXNOD) GO TO 910
      X(KK) = 0
      SORTIE = .FALSE.
  187 CONTINUE
      NARC = NARC + 1
      FROM(NARC) = FINII
      TO(NARC) = KK
      H(NARC) = 1
      C(NARC) = (IABS(D(I)) - FINHR(TII))*MULT
      GO TO 195
*
  190 CONTINUE
      NARC = NARC + 1
      IF (NARC.GT.MAXARC) GO TO 900
      FROM(NARC) = KK
      TO(NARC) = DEBII
      H(NARC) = 1
      C(NARC) = (DEBHR(TII) - D(I))*MULT
      SORTIE = .TRUE.
*
  195 CONTINUE
      GO TO 180
*
*   PUT IN RETURN ARC
*
  200 CONTINUE
      NARC = NARC + 1
      IF(NARC.GT.MAXARC) GO TO 900
      FROM(NARC) = KK
      TO(NARC) = K + 1
      C(NARC) = AMENDE
      H(NARC) = NVOY
*
*   SOLVE TRANSHIPMENT PROBLEM
*
      N = NARC
      M = KK
      IOUT = 6
      CALL RNFT
      CALL RNWFPA(IOUT)
*     CALL RNWSOL(IOUT,2,2,1,1,N,1,M,CODE)
      IF (RTN(1).NE.0) GO TO 910
```

```
*
*   SET UP SUCCESSORS, PREDECESSORS BASED ON RNET SOLUTION
*
      DO 250 II = 1,NVOY
      FINII = 2*II
      DEBII = FINII - 1
      IF (X(DEBII).EQ.0) GO TO 225
      NUM = IABS(F(DEBII))
      IF (NUM.GE.K) GO TO 225
      NUM = NUM/2
      DERNII(II) = NUM
      PRCHII(NUM) = II
  225 CONTINUE
      IF (X(FINII).EQ.0) GO TO 250
      NUM = IABS(F(FINII))
      IF (NUM.GE.K) GO TO 250
      NUM = (NUM + 1)/2
      PRCHII(II) = NUM
      DERNII(NUM) = II
  250 CONTINUE
*
      DO 275 I = 1,N
      IF (H(I).GE.0) GO TO 275
      FNUM = FROM(I)/2
      TNUM = (TO(I) + 1)/2
      IF (FROM(I).GE.K) FNUM = 0
      IF (TO(I).GE.K) TNUM = 0
      IF (FNUM.GT.0) PRCHII(FNUM) = TNUM
      IF (TNUM.GT.0) DERNII(TNUM) = FNUM
  275 CONTINUE
*
      DO 300 II = 1,NVOY
      IF (DERNII(II).NE.0) GO TO 300
      GARSOR = SORA
      IF (DEBEN(II).EQ.BNODE) GARSOR = SORB
      BPER = (DERHR(II) - GARSOR - SHIFT - REP)/PERLEN + 1
      KK = II
  290 CONTINUE
      B(KK) = BPER
      IF (PRCHII(KK).EQ.0) GO TO 300
      KK = PRCHII(KK)
      GO TO 290
  300 CONTINUE
*
*   WRITE REPORT
*
      WRITE(6,305)
  305 FORMAT('1MIN-COST BLOCK SOLUTION'//' ')
      WRITE(8,310) ANODE,SORA,RENA
      WRITE(8,310) BNODE,SORB,RENB
      WRITE(8,310) PERLEN,SHIFT
  310 FORMAT(5I4)
*
      DO 400 II = 1,NVOY
*
      DEB = DEBHR(II)
      DEB = DEB + (DEB/60)*40
      FIN = FINHR(II)
      FIN = FIN + (FIN/60)*40
      GARREN = RENA
      IF (FINFN(II).EQ.BNODE) GARREN = RENB
```

```
      ZPER = (FINHR(II) + OARREN - SHIFT + REP)/PERLEN
*     WRITE(6,351) FIN,FINHR(II),OARREN,SHIFT,PERLEN,ZPER
* 351 FORMAT(' FIN = ',I4,' FINHR = ',I4,' OARREN = ',I3,
*    ' SHIFT = ',I3,' PERLEN = ',I3,' ZPER = ',I3)
      WRITE(6,355) II,DERNII(II),DEBEN(II),DEB,
     FIN,FINEN(II),PRCHII(II),B(II),ZPER
  355 FORMAT(' ',I4,I5,I6,3I5,I6,2I5)
      WRITE(8,365) DEB,FIN,DEBEN(II),FINEN(II),
     PRCHII(II)
  365 FORMAT(5I4)
  400 CONTINUE
*
      WRITE(8,405) ORONUM
  405 FORMAT(I4)
      GO TO 999
*
  900 CONTINUE
      WRITE(6,905) II
  905 FORMAT('OARC OVERFLOW,  II = ',I5)
      GO TO 999
  910 CONTINUE
      WRITE(6,915)
  915 FORMAT('ONODE OVERFLOW')
*
  999 CONTINUE
      STOP
      END
```

```
        INTEGER FUNCTION CONV(TIME)
*
*     THIS FUNCTION CONVERTS TIMES IN HOURS AND MINUTES
*     INTO TIMES IN MINUTES
*
        IMPLICIT INTEGER(A-Z)
*
        HR = TIME/100
        MIN = TIME - HR*100
        CONV = 60*HR + MIN
*
        RETURN
        END
```

Appendix D

Trip timetable data from Montréal

TABLE D-1

TRIP DATA FOR CTCUM LINE 32

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
1	0	4:25/210	4:58/209	4
2	0	4:40/210	5:13/209	7
3	0	4:54/210	5:27/209	10
4	1	4:58/209	5:36/210	11
5	0	5:08/210	5:41/209	13
6	0	5:16/210	5:49/209	15
7	2	5:13/209	5:51/210	16
8	0	5:23/210	5:56/209	17
9	0	5:31/210	6:04/209	19
10	3	5:27/209	6:05/210	20
11	4	5:37/210	6:10/209	21
12	0	5:43/210	6:16/209	23
13	5	5:41/209	6:19/210	24
14	0	5:49/210	6:23/209	25
15	6	5:49/209	6:27/210	27
16	7	5:55/210	6:30/209	28
17	8	5:56/209	6:34/210	30
18	0	6:02/210	6:37/209	29
19	9	6:04/209	6:42/210	31
20	10	6:09/210	6:44/209	32
21	11	6:10/209	6:49/210	33
22	0	6:14/210	6:50/209	34
23	12	6:16/209	6:57/210	37
24	13	6:20/210	6:57/209	36
25	14	6:23/209	7:04/210	39
26	0	6:26/210	7:04/209	38
27	15	6:32/210	7:11/209	40
28	16	6:30/209	7:12/210	42
29	18	6:37/209	7:19/210	45
30	17	6:38/210	7:19/209	43
31	19	6:44/210	7:26/209	46
32	20	6:44/209	7:27/210	47
33	21	6:50/210	7:34/209	49
34	22	6:50/209	7:34/210	50
35	0	6:56/210	7:41/209	52
36	24	6:57/209	7:42/210	53
37	23	7:02/210	7:49/209	55
38	26	7:04/209	7:49/210	56
39	25	7:09/210	7:56/209	59
40	27	7:11/209	7:56/210	58
41	0	7:15/209	8:00/210	0
42	28	7:16/210	8:03/209	61
43	30	7:19/209	8:04/210	60
44	0	7:23/209	8:08/210	62
45	29	7:22/210	8:09/209	63
46	31	7:27/209	8:12/210	0
47	32	7:29/210	8:16/209	65
48	0	7:31/209	8:16/210	64
49	33	7:35/209	8:20/210	0
50	34	7:35/210	8:22/209	67
51	0	7:39/209	8:24/210	66
52	35	7:43/209	8:28/210	0
53	36	7:42/210	8:29/209	68
54	0	7:47/209	8:32/210	69
55	37	7:51/209	8:36/210	0
56	38	7:49/210	8:36/209	70

TABLE D-1 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
57	0	7:55/209	8:40/210	71
58	40	7:56/210	8:43/209	72
59	39	7:59/209	8:44/210	73
60	43	8:04/210	8:49/209	74
61	42	8:05/209	8:50/210	0
62	44	8:11/210	8:55/209	76
63	45	8:11/209	8:56/210	75
64	48	8:18/210	9:01/209	78
65	47	8:17/209	9:02/210	77
66	51	8:26/210	9:07/209	0
67	50	8:23/209	9:08/210	79
68	53	8:29/209	9:14/210	81
69	54	8:33/210	9:14/209	80
70	56	8:36/209	9:21/210	83
71	57	8:41/210	9:22/209	82
72	58	8:44/209	9:29/210	84
73	59	8:48/210	9:29/209	85
74	60	8:51/209	9:35/210	0
75	63	8:56/210	9:37/209	0
76	62	8:59/209	9:41/210	86
77	65	9:03/210	9:44/209	87
78	64	9:06/209	9:48/210	88
79	67	9:11/210	9:52/209	90
80	69	9:14/209	9:56/210	89
81	68	9:18/210	9:59/209	91
82	71	9:24/209	10:06/210	92
83	70	9:26/210	10:07/209	0
84	72	9:33/210	10:14/209	94
85	73	9:34/209	10:16/210	93
86	76	9:41/210	10:22/209	96
87	77	9:45/209	10:27/210	95
88	78	9:48/210	10:29/209	0
89	80	9:56/210	10:37/209	98
90	79	9:55/209	10:37/210	97
91	81	10:05/209	10:47/210	99
92	82	10:06/210	10:47/209	100
93	85	10:16/210	10:57/209	102
94	84	10:16/209	10:58/210	101
95	87	10:27/210	11:08/209	104
96	86	10:26/209	11:08/210	103
97	90	10:37/210	11:18/209	106
98	89	10:37/209	11:19/210	105
99	91	10:47/210	11:28/209	108
100	92	10:47/209	11:29/210	107
101	94	10:58/210	11:39/209	111
102	93	10:57/209	11:39/210	109
103	96	11:08/210	11:49/209	113
104	95	11:08/209	11:50/210	112
105	98	11:19/210	12:00/209	115
106	97	11:18/209	12:01/210	114
107	100	11:29/210	12:10/209	117
108	99	11:28/209	12:12/210	116
109	102	11:39/210	12:20/209	119
110	0	11:37/209	12:21/210	118
111	101	11:45/209	12:29/210	120
112	104	11:50/210	12:31/209	123

TARLE D-1 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
113	103	11:54/209	12:38/210	122
114	106	12:01/210	12:42/209	125
115	105	12:02/209	12:46/210	124
116	108	12:12/210	12:53/209	127
117	107	12:11/209	12:55/210	126
118	110	12:21/210	13:02/209	129
119	109	12:20/209	13:04/210	128
120	111	12:29/210	13:10/209	131
121	0	12:28/209	13:12/210	130
122	113	12:38/210	13:19/209	133
123	112	12:36/209	13:20/210	132
124	115	12:46/210	13:27/209	135
125	114	12:45/209	13:29/210	134
126	117	12:55/210	13:36/209	137
127	116	12:53/209	13:37/210	136
128	119	13:04/210	13:45/209	139
129	118	13:02/209	13:46/210	138
130	121	13:12/210	13:53/209	141
131	120	13:10/209	13:54/210	140
132	123	13:20/210	14:01/209	143
133	122	13:19/209	14:03/210	142
134	125	13:29/210	14:10/209	145
135	124	13:27/209	14:11/210	144
136	127	13:37/210	14:18/209	147
137	126	13:36/209	14:20/210	148
138	129	13:46/210	14:27/209	149
139	128	13:45/209	14:29/210	150
140	131	13:54/210	14:35/209	152
141	130	13:53/209	14:37/210	153
142	133	14:03/210	14:44/209	154
143	132	14:02/209	14:46/210	155
144	135	14:11/210	14:52/209	159
145	134	14:10/209	14:54/210	158
146	0	14:18/210	14:59/209	161
147	136	14:18/209	15:03/210	163
148	137	14:25/210	15:06/209	164
149	138	14:27/209	15:12/210	167
150	139	14:30/210	15:14/209	168
151	0	14:35/210	15:20/209	171
152	140	14:35/209	15:21/210	172
153	141	14:40/210	15:25/209	173
154	142	14:44/209	15:30/210	176
155	143	14:46/210	15:31/209	175
156	0	14:50/210	15:36/209	177
157	0	14:50/209	15:36/210	178
158	145	14:54/210	15:40/209	180
159	144	14:55/209	15:41/210	179
160	0	14:58/210	15:44/209	181
161	146	15:00/209	15:46/210	182
162	0	15:02/210	15:48/209	0
163	147	15:05/210	15:51/209	183
164	148	15:06/209	15:52/210	184
165	0	15:09/210	15:55/209	185
166	0	15:11/209	15:57/210	186
167	149	15:13/210	15:59/209	187
168	150	15:16/209	16:02/210	188

TABLE D-1 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
169	0	15:17/210	16:03/209	0
170	0	15:20/210	16:06/209	189
171	151	15:21/209	16:07/210	190
172	152	15:24/210	16:10/209	191
173	153	15:26/209	16:12/210	192
174	0	15:28/210	16:14/209	193
175	155	15:31/209	16:17/210	194
176	154	15:32/210	16:18/209	0
177	156	15:36/209	16:22/210	195
178	157	15:36/210	16:22/209	196
179	159	15:41/210	16:27/209	197
180	158	15:41/209	16:27/210	198
181	160	15:46/209	16:32/210	199
182	161	15:46/210	16:32/209	200
183	163	15:51/209	16:37/210	201
184	164	15:52/210	16:38/209	202
185	165	15:56/209	16:42/210	203
186	166	15:57/210	16:43/209	204
187	167	16:01/209	16:47/210	205
188	168	16:02/210	16:48/209	206
189	170	16:06/209	16:52/210	207
190	171	16:07/210	16:53/209	208
191	172	16:11/209	16:57/210	209
192	173	16:12/210	16:58/209	210
193	174	16:16/209	17:02/210	211
194	175	16:17/210	17:03/209	212
195	177	16:22/210	17:08/209	213
196	178	16:22/209	17:08/210	214
197	179	16:27/209	17:13/210	215
198	180	16:27/210	17:13/209	216
199	181	16:32/210	17:18/209	217
200	182	16:32/209	17:18/210	218
201	183	16:37/210	17:23/209	219
202	184	16:38/209	17:24/210	220
203	185	16:42/210	17:28/209	221
204	186	16:43/209	17:29/210	222
205	187	16:47/210	17:33/209	223
206	188	16:48/209	17:34/210	224
207	189	16:52/210	17:38/209	225
208	190	16:53/209	17:39/210	226
209	191	16:57/210	17:43/209	227
210	192	16:58/209	17:44/210	228
211	193	17:02/210	17:48/209	229
212	194	17:03/209	17:49/210	230
213	195	17:08/209	17:54/210	232
214	196	17:08/210	17:54/209	231
215	197	17:13/210	17:59/209	0
216	198	17:13/209	17:59/210	233
217	199	17:18/209	18:04/210	234
218	200	17:18/210	18:04/209	235
219	201	17:23/209	18:09/210	236
220	202	17:24/210	18:10/209	0
221	203	17:28/209	18:14/210	237
222	204	17:29/210	18:15/209	238
223	205	17:34/209	18:18/210	239
224	206	17:34/210	18:20/209	0

TARLE D-1 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
225	207	17:40/209	18:22/210	240
226	208	17:39/210	18:25/209	242
227	209	17:46/209	18:27/210	241
228	210	17:44/210	18:30/209	0
229	211	17:52/209	18:32/210	243
230	212	17:49/210	18:35/209	0
231	214	17:58/209	18:38/210	245
232	213	17:54/210	18:39/209	244
233	216	17:59/210	18:43/209	0
234	217	18:04/210	18:47/209	246
235	218	18:08/209	18:48/210	247
236	219	18:09/210	18:51/209	0
237	221	18:14/210	18:55/209	0
238	222	18:18/209	18:58/210	249
239	223	18:18/210	18:59/209	248
240	225	18:22/210	19:03/209	0
241	227	18:27/210	19:08/209	250
242	226	18:28/209	19:08/210	251
243	229	18:32/210	19:13/209	0
244	232	18:39/209	19:17/210	253
245	231	18:38/210	19:19/209	252
246	234	18:49/209	19:27/210	255
247	235	18:48/210	19:29/209	254
248	239	18:59/209	19:37/210	257
249	238	18:58/210	19:39/209	256
250	241	19:09/209	19:47/210	259
251	242	19:08/210	19:49/209	258
252	245	19:19/209	19:57/210	261
253	244	19:17/210	19:58/209	260
254	247	19:29/209	20:07/210	262
255	246	19:27/210	20:08/209	263
256	249	19:39/209	20:17/210	265
257	248	19:37/210	20:18/209	264
258	251	19:49/209	20:27/210	267
259	250	19:47/210	20:27/209	266
260	253	19:58/209	20:36/210	269
261	252	19:57/210	20:36/209	268
262	254	20:07/210	20:45/209	270
263	255	20:08/209	20:45/210	271
264	257	20:18/209	20:54/210	273
265	256	20:17/210	20:55/209	272
266	259	20:27/209	21:03/210	275
267	258	20:27/210	21:05/209	274
268	261	20:36/209	21:12/210	277
269	260	20:36/210	21:14/209	276
270	262	20:45/209	21:21/210	278
271	263	20:45/210	21:23/209	279
272	265	20:55/209	21:31/210	280
273	264	20:54/210	21:32/209	281
274	267	21:05/209	21:41/210	282
275	266	21:03/210	21:41/209	0
276	269	21:14/209	21:50/210	283
277	268	21:12/210	21:50/209	284
278	270	21:21/210	21:59/209	286
279	271	21:24/209	22:00/210	285
280	272	21:31/210	22:09/209	0

TARLE D-1 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
281	273	21:38/209	22:14/210	287
282	274	21:41/210	22:19/209	288
283	276	21:50/210	22:28/209	0
284	277	21:53/209	22:29/210	289
285	279	22:00/210	22:38/209	290
286	278	22:08/209	22:44/210	291
287	281	22:14/210	22:52/209	292
288	282	22:23/209	22:59/210	293
289	284	22:29/210	23:07/209	294
290	285	22:38/209	23:14/210	295
291	286	22:44/210	23:22/209	296
292	287	22:53/209	23:29/210	297
293	288	22:59/210	23:37/209	298
294	289	23:08/209	23:44/210	299
295	290	23:14/210	23:52/209	300
296	291	23:23/209	23:59/210	301
297	292	23:29/210	24:07/209	302
298	293	23:38/209	24:14/210	303
299	294	23:44/210	24:22/209	304
300	295	23:53/209	24:29/210	305
301	296	23:59/210	24:37/209	307
302	297	24:11/209	24:47/210	306
303	298	24:14/210	24:51/209	309
304	299	24:28/209	25:04/210	308
305	300	24:29/210	25:05/209	0
306	302	24:47/210	25:20/209	311
307	301	24:45/209	25:21/210	310
308	304	25:04/210	25:37/209	0
309	303	25:02/209	25:38/210	312
310	307	25:21/210	25:54/209	0
311	306	25:20/209	25:55/210	313
312	309	25:38/210	26:11/209	0
313	311	25:55/210	26:28/209	0

GARAGE DEADHEAD TIMES

FROM ANJOU GARAGE TO POINT 209: 15

FROM POINT 209 TO ANJOU GARAGE: 13

FROM ANJOU GARAGE TO POINT 210: 17

FROM POINT 210 TO ANJOU GARAGE: 16

TABLE D-2

TRIP DATA FOR CTCUM LINE 139

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
1	0	4:55/739	5:35/740	8
2	0	5:10/740	5:49/739	10
3	0	5:10/739	5:50/740	11
4	0	5:25/739	6:05/740	15
5	0	5:30/740	6:09/739	17
6	0	5:33/739	6:13/740	18
7	0	5:38/739	6:18/740	20
8	1	5:42/740	6:21/739	22
9	0	5:44/739	6:24/740	23
10	2	5:49/739	6:29/740	25
11	3	5:54/740	6:33/739	28
12	0	5:53/739	6:33/740	27
13	0	5:57/739	6:37/740	29
14	0	6:01/739	6:41/740	31
15	4	6:05/740	6:44/739	32
16	0	6:06/739	6:46/740	34
17	5	6:10/739	6:50/740	36
18	6	6:13/740	6:52/739	37
19	0	6:14/739	6:55/740	39
20	7	6:18/740	6:59/739	40
21	0	6:18/739	7:00/740	42
22	8	6:22/739	7:05/740	45
23	9	6:24/740	7:07/739	46
24	0	6:27/739	7:10/740	47
25	10	6:29/740	7:12/739	49
26	0	6:32/739	7:15/740	50
27	12	6:33/740	7:17/739	51
28	11	6:37/739	7:20/740	55
29	13	6:37/740	7:22/739	54
30	0	6:42/739	7:25/740	58
31	14	6:41/740	7:27/739	57
32	15	6:45/739	7:28/740	60
33	0	6:48/739	7:31/740	62
34	16	6:46/740	7:32/739	61
35	0	6:51/739	7:34/740	63
36	17	6:50/740	7:37/739	65
37	18	6:54/739	7:37/740	66
38	0	6:57/739	7:40/740	67
39	19	6:55/740	7:42/739	70
40	20	7:00/739	7:43/740	69
41	0	7:02/739	7:45/740	72
42	21	7:00/740	7:47/739	71
43	0	7:04/739	7:48/740	74
44	0	7:06/739	7:51/740	0
45	22	7:05/740	7:52/739	75
46	23	7:08/739	7:54/740	77
47	24	7:10/740	7:57/739	78
48	0	7:11/739	7:57/740	0
49	25	7:14/739	8:00/740	80
50	26	7:15/740	8:02/739	79
51	27	7:17/739	8:03/740	0
52	0	7:19/739	8:05/740	82
53	0	7:19/740	8:06/739	81
54	29	7:22/739	8:08/740	0
55	28	7:23/740	8:10/739	83
56	0	7:25/739	8:11/740	85

TABLE D-2 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
57	31	7:27/739	8:13/740	0
58	30	7:27/740	8:14/739	84
59	0	7:30/739	8:16/740	0
60	32	7:30/740	8:17/739	86
61	34	7:33/739	8:19/740	88
62	33	7:32/740	8:19/739	87
63	35	7:35/740	8:22/739	0
64	0	7:36/739	8:22/740	0
65	36	7:38/739	8:24/740	90
66	37	7:37/740	8:24/739	89
67	38	7:40/740	8:27/739	91
68	0	7:41/739	8:27/740	0
69	40	7:43/740	8:30/739	0
70	39	7:44/739	8:30/740	0
71	42	7:47/739	8:32/740	93
72	41	7:45/740	8:32/739	92
73	0	7:50/739	8:34/740	0
74	43	7:48/740	8:35/739	94
75	45	7:53/739	8:36/740	0
76	0	7:56/739	8:39/740	95
77	46	7:54/740	8:41/739	96
78	47	7:59/739	8:42/740	0
79	50	8:02/739	8:45/740	98
80	49	8:00/740	8:47/739	97
81	53	8:06/739	8:49/740	0
82	52	8:06/740	8:53/739	99
83	55	8:10/739	8:53/740	100
84	58	8:14/739	8:57/740	0
85	56	8:12/740	8:59/739	101
86	60	8:17/739	9:00/740	102
87	62	8:21/739	9:04/740	104
88	61	8:19/740	9:05/739	103
89	66	8:25/739	9:08/740	0
90	65	8:25/740	9:10/739	105
91	67	8:29/739	9:12/740	106
92	72	8:32/739	9:15/740	0
93	71	8:32/740	9:17/739	107
94	74	8:36/739	9:19/740	108
95	76	8:39/740	9:24/739	109
96	77	8:41/739	9:24/740	110
97	80	8:47/739	9:30/740	112
98	79	8:46/740	9:31/739	111
99	82	8:53/739	9:36/740	114
100	83	8:53/740	9:38/739	113
101	85	8:59/739	9:42/740	116
102	86	9:00/740	9:45/739	115
103	88	9:05/739	9:48/740	0
104	87	9:06/740	9:51/739	117
105	90	9:11/739	9:54/740	118
106	91	9:13/740	9:58/739	119
107	93	9:17/739	10:00/740	120
108	94	9:20/740	10:05/739	121
109	95	9:24/739	10:07/740	122
110	96	9:26/740	10:11/739	123
111	98	9:31/739	10:14/740	124
112	97	9:33/740	10:18/739	125

TABLE D-2 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
113	100	9:38/739	10:21/740	126
114	99	9:40/740	10:25/739	127
115	102	9:45/739	10:28/740	128
116	101	9:47/740	10:32/739	129
117	104	9:51/739	10:34/740	130
118	105	9:54/740	10:39/739	131
119	106	9:58/739	10:41/740	132
120	107	10:00/740	10:45/739	133
121	108	10:05/739	10:48/740	134
122	109	10:07/740	10:52/739	135
123	110	10:11/739	10:54/740	136
124	111	10:14/740	10:59/739	137
125	112	10:18/739	11:01/740	138
126	113	10:21/740	11:06/739	141
127	114	10:25/739	11:08/740	140
128	115	10:28/740	11:13/739	143
129	116	10:32/739	11:15/740	142
130	117	10:34/740	11:19/739	145
131	118	10:39/739	11:22/740	144
132	119	10:41/740	11:26/739	147
133	120	10:45/739	11:28/740	146
134	121	10:48/740	11:33/739	150
135	122	10:52/739	11:35/740	149
136	123	10:54/740	11:39/739	152
137	124	10:59/739	11:42/740	151
138	125	11:01/740	11:46/739	154
139	0	11:04/739	11:48/740	153
140	127	11:08/740	11:53/739	156
141	126	11:09/739	11:54/740	155
142	129	11:15/740	12:00/739	157
143	128	11:14/739	12:00/740	158
144	131	11:22/740	12:07/739	161
145	130	11:20/739	12:07/740	160
146	133	11:28/740	12:13/739	163
147	132	11:26/739	12:13/740	162
148	0	11:31/739	12:18/740	164
149	135	11:35/740	12:20/739	165
150	134	11:37/739	12:24/740	166
151	137	11:42/740	12:27/739	167
152	136	11:43/739	12:30/740	168
153	139	11:48/740	12:34/739	169
154	138	11:48/739	12:35/740	170
155	141	11:54/740	12:41/739	173
156	140	11:54/739	12:41/740	172
157	142	12:00/739	12:47/740	174
158	143	12:00/740	12:48/739	175
159	0	12:05/739	12:52/740	176
160	145	12:07/740	12:55/739	177
161	144	12:11/739	12:58/740	178
162	147	12:13/740	13:02/739	179
163	146	12:17/739	13:04/740	180
164	148	12:18/740	13:07/739	181
165	149	12:22/739	13:09/740	182
166	150	12:24/740	13:13/739	183
167	151	12:28/739	13:15/740	184
168	152	12:30/740	13:19/739	185

TARLE D-2 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
169	153	12:34/739	13:21/740	186
170	154	12:35/740	13:24/739	187
171	0	12:39/739	13:26/740	188
172	156	12:41/740	13:30/739	189
173	155	12:45/739	13:32/740	190
174	157	12:47/740	13:36/739	191
175	158	12:50/739	13:37/740	192
176	159	12:52/740	13:41/739	193
177	160	12:56/739	13:43/740	194
178	161	12:58/740	13:47/739	195
179	162	13:02/739	13:49/740	196
180	163	13:04/740	13:53/739	197
181	164	13:07/739	13:54/740	198
182	165	13:09/740	13:58/739	199
183	166	13.13/739	14:00/740	200
184	167	13:15/740	14:04/739	201
185	168	13:19/739	14:06/740	202
186	169	13:21/740	14:10/739	203
187	170	13:24/739	14:11/740	204
188	171	13:26/740	14:15/739	205
189	172	13:30/739	14:17/740	206
190	173	13:32/740	14:21/739	207
191	174	13:36/739	14:23/740	210
192	175	13:37/740	14:26/739	209
193	176	13:41/739	14:28/740	212
194	177	13:43/740	14:32/739	211
195	178	13:47/739	14:34/740	214
196	179	13:49/740	14:38/739	213
197	180	13:53/739	14:40/740	218
198	181	13:54/740	14:43/739	215
199	182	13:58/739	14:45/740	220
200	183	14:00/740	14:49/739	217
201	184	14:04/739	14:51/740	221
202	185	14:06/740	14:55/739	219
203	186	14:10/739	14:57/740	223
204	187	14:11/740	15:00/739	222
205	188	14:15/739	15:02/740	225
206	189	14:17/740	15:06/739	224
207	190	14:21/739	15:08/740	229
208	0	14:22/740	15:11/739	226
209	192	14:26/739	15:13/740	231
210	191	14:27/740	15:16/739	228
211	194	14:32/739	15:19/740	235
212	193	14:31/740	15:21/739	233
213	196	14:38/739	15:25/740	238
214	195	14:35/740	15:26/739	234
215	198	14:43/739	15:30/740	241
216	0	14:39/740	15:31/739	237
217	200	14:49/739	15:36/740	244
218	197	14:43/740	15:37/739	240
219	202	14:55/739	15:42/740	248
220	199	14:48/740	15:42/739	245
221	201	14:53/740	15:47/739	247
222	204	15:00/739	15:47/740	250
223	203	14:58/740	15:52/739	251
224	206	15:06/739	15:53/740	254

TABLE D-2 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
225	205	15:03/740	15:57/739	253
226	208	15:11/739	15:58/740	259
227	0	15:07/740	16:01/739	255
228	210	15:16/739	16:03/740	260
229	207	15:10/740	16:04/739	256
230	0	15:20/739	16:07/740	263
231	209	15:13/740	16:07/739	258
232	0	15:17/740	16:11/739	261
233	212	15:24/739	16:11/740	266
234	214	15:27/739	16:14/740	269
235	211	15:20/740	16:14/739	262
236	0	15:23/740	16:17/739	264
237	216	15:31/739	16:18/740	270
238	213	15:27/740	16:21/739	267
239	0	15:34/739	16:21/740	273
240	218	15:37/739	16:24/740	275
241	215	15:30/740	16:24/739	268
242	0	15:33/740	16:27/739	271
243	0	15:41/739	16:28/740	277
244	217	15:37/740	16:31/739	272
245	220	15:44/739	16:31/740	278
246	0	15:40/740	16:34/739	274
247	221	15:47/739	16:34/740	280
248	219	15:43/740	16:37/739	276
249	0	15:51/739	16:38/740	284
250	222	15:47/740	16:41/739	279
251	223	15:54/739	16:41/740	285
252	0	15:50/740	16:44/739	282
253	225	15:57/739	16:44/740	287
254	224	15:53/740	16:47/739	283
255	227	16:01/739	16:48/740	291
256	229	16:04/739	16:51/740	292
257	0	15:57/740	16:51/739	286
258	231	16:07/739	16:54/740	294
259	226	16:00/740	16:54/739	288
260	228	16:03/740	16:57/739	290
261	232	16:11/739	16:58/740	298
262	235	16:14/739	17:01/740	299
263	230	16:07/740	17:01/739	293
264	236	16:17/739	17:04/740	301
265	0	16:10/740	17:04/739	295
266	233	16:13/740	17:07/739	297
267	238	16:21/739	17:08/740	305
268	241	16:24/739	17:11/740	306
269	234	16:17/740	17:11/739	300
270	237	16:20/740	17:14/739	302
271	242	16:27/739	17:14/740	309
272	244	16:31/739	17:16/740	311
273	239	16:23/740	17:17/739	304
274	246	16:34/739	17:19/740	313
275	240	16:26/740	17:20/739	307
276	248	16:37/739	17:22/740	315
277	243	16:28/740	17:22/739	308
278	245	16:31/740	17:24/739	310
279	250	16:41/739	17:25/740	316
280	247	16:34/740	17:26/739	312

TABLE D-2 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
281	0	16:36/740	17:28/739	314
282	252	16:44/739	17:28/740	318
283	254	16:47/739	17:31/740	320
284	249	16:39/740	17:31/739	317
285	251	16:42/740	17:34/739	319
286	257	16:51/739	17:35/740	323
287	253	16:45/740	17:37/739	321
288	259	16:54/739	17:38/740	326
289	0	16:47/740	17:39/739	322
290	260	16:57/739	17:41/740	329
291	255	16:50/740	17:42/739	324
292	256	16:52/740	17:44/739	325
293	263	17:01/739	17:45/740	0
294	258	16:55/740	17:47/739	327
295	265	17:04/739	17:48/740	331
296	0	16:57/740	17:49/739	328
297	266	17:07/739	17:51/740	334
298	261	17:00/740	17:52/739	330
299	262	17:02/740	17:54/739	332
300	269	17:11/739	17:55/740	336
301	264	17:05/740	17:57/739	333
302	270	17:14/739	17:58/740	0
303	0	17:07/740	17:59/739	335
304	273	17:17/739	18:01/740	339
305	267	17:10/740	18:02/739	337
306	268	17:12/740	18:04/739	338
307	275	17:20/739	18:04/740	342
308	277	17:22/739	18:06/740	0
309	271	17:15/740	18:07/739	340
310	278	17:24/739	18:08/740	0
311	272	17:17/740	18:09/739	341
312	280	17:26/739	18:10/740	345
313	274	17:20/740	18:11/739	343
314	281	17:28/739	18:12/740	347
315	276	17:23/740	18:13/739	344
316	279	17:26/740	18:15/739	346
317	284	17:31/739	18:15/740	0
318	282	17:29/740	18:18/739	0
319	285	17:34/739	18:18/740	349
320	283	17:32/740	18:21/739	348
321	287	17:37/739	18:21/740	0
322	289	17:39/739	18:23/740	351
323	286	17:35/740	18:24/739	350
324	291	17:42/739	18:26/740	352
325	292	17:44/739	18:28/740	0
326	288	17:40/740	18:29/739	0
327	294	17:47/739	18:31/740	354
328	296	17:49/739	18:33/740	355
329	290	17:44/740	18:33/739	353
330	298	17:52/739	18:36/740	0
331	295	17:48/740	18:37/739	356
332	299	17:54/739	18:38/740	357
333	301	17:57/739	18:41/740	0
334	297	17:53/740	18:42/739	0
335	303	17:59/739	18:43/740	359
336	300	17:57/740	18:46/739	358

TABLE D-2 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
337	305	18:02/739	18:46/740	0
338	306	18:04/739	18:48/740	360
339	304	18:01/740	18:50/739	0
340	309	18:07/739	18:51/740	0
341	311	18:09/739	18:53/740	362
342	307	18:05/740	18:54/739	361
343	313	18:11/739	18:55/740	0
344	315	18:13/739	18:57/740	363
345	312	18:10/740	18:59/739	364
346	316	18:15/739	18:59/740	365
347	314	18:14/740	19:03/739	0
348	320	18:21/739	19:05/740	367
349	319	18:18/740	19:07/739	366
350	323	18:28/739	19:12/740	369
351	322	18:23/740	19:12/739	0
352	324	18:27/740	19:16/739	368
353	329	18:35/739	19:19/740	371
354	327	18:31/740	19:20/739	370
355	328	18:35/740	19:24/739	0
356	331	18:41/739	19:25/740	373
357	332	18:40/740	19:29/739	372
358	336	18:48/739	19:32/740	375
359	335	18:44/740	19:33/739	374
360	338	18:48/740	19:37/739	0
361	342	18:55/739	19:39/740	376
362	341	18:53/740	19:42/739	377
363	344	18:57/740	19:46/739	0
364	345	19:02/739	19:46/740	378
365	346	19:01/740	19:50/739	379
366	349	19:09/739	19:53/740	380
367	348	19:05/740	19:54/739	381
368	352	19:16/739	19:59/740	382
369	350	19:12/740	20:01/739	383
370	354	19:23/739	20:05/740	384
371	353	19:19/740	20:08/739	385
372	357	19:29/739	20:11/740	386
373	356	19:25/740	20:14/739	387
374	359	19:36/739	20:18/740	388
375	358	19:32/740	20:19/739	0
376	361	19:39/740	20:24/739	389
377	362	19:43/739	20:25/740	390
378	364	19:46/740	20:31/739	391
379	365	19:50/739	20:32/740	392
380	366	19:53/740	20:38/739	393
381	367	19:57/739	20:39/740	394
382	368	19:59/740	20:44/739	395
383	369	20:04/739	20:46/740	396
384	370	20:05/740	20:50/739	397
385	371	20:11/739	20:53/740	398
386	372	20:11/740	20:56/739	399
387	373	20:18/739	21:00/740	400
388	374	20:18/740	21:03/739	401
389	376	20:24/739	21:06/740	402
390	377	20:25/740	21:10/739	0
391	378	20:31/739	21:13/740	403
392	379	20:32/740	21:17/739	404

TABLE D-2 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
393	380	20: 38/739	21: 19/740	405
394	381	20: 39/740	21: 24/739	406
395	382	20: 45/739	21: 25/740	407
396	383	20: 46/740	21: 31/739	0
397	384	20: 52/739	21: 32/740	408
398	385	20: 53/740	21: 38/739	409
399	386	21: 00/739	21: 40/740	410
400	387	21: 00/740	21: 45/739	411
401	388	21: 09/739	21: 49/740	412
402	389	21: 06/740	21: 51/739	0
403	391	21: 13/740	21: 58/739	413
404	392	21: 18/739	21: 58/740	414
405	393	21: 19/740	22: 04/739	415
406	394	21: 28/739	22: 08/740	416
407	395	21: 25/740	22: 10/739	0
408	397	21: 32/740	22: 17/739	417
409	398	21: 38/739	22: 18/740	418
410	399	21: 40/740	22: 25/739	420
411	400	21: 48/739	22: 28/740	419
412	401	21: 49/740	22: 33/739	421
413	403	21: 58/739	22: 38/740	422
414	404	21: 58/740	22: 40/739	0
415	405	22: 08/739	22: 48/740	424
416	406	22: 08/740	22: 48/739	423
417	408	22: 18/739	22: 58/740	426
418	409	22: 16/740	22: 58/739	425
419	411	22: 28/740	23: 08/739	427
420	410	22: 28/739	23: 08/740	428
421	412	22: 38/739	23: 18/740	430
422	413	22: 38/740	23: 18/739	429
423	416	22: 48/739	23: 27/740	432
424	415	22: 48/740	23: 28/739	431
425	418	22: 58/739	23: 36/740	434
426	417	22: 58/740	23: 38/739	433
427	419	23: 08/739	23: 45/740	435
428	420	23: 08/740	23: 48/739	436
429	422	23: 18/739	23: 55/740	438
430	421	23: 18/740	23: 58/739	437
431	424	23: 28/739	24: 05/740	439
432	423	23: 27/740	24: 07/739	440
433	426	23: 38/739	24: 15/740	441
434	425	23: 36/740	24: 16/739	442
435	427	23: 45/740	24: 25/739	444
436	428	23: 48/739	24: 25/740	443
437	430	23: 58/739	24: 35/740	445
438	429	23: 55/740	24: 35/739	0
439	431	24: 05/740	24: 45/739	447
440	432	24: 08/739	24: 45/740	446
441	433	24: 15/740	24: 55/739	449
442	434	24: 21/739	24: 57/740	448
443	436	24: 25/740	25: 04/739	0
444	435	24: 34/739	25: 09/740	450
445	437	24: 35/740	25: 13/739	451
446	440	24: 45/740	25: 21/739	0
447	439	24: 47/739	25: 21/740	452
448	442	24: 57/740	25: 30/739	453

TARLE D-2 (CONTINUED)

I	PR(I)	START TIME/LOC	END TIME/LOC	SU(I)
449	441	25:00/739	25:33/740	454
450	444	25:09/740	25:40/739	0
451	445	25:15/739	25:45/740	455
452	447	25:21/740	25:52/739	0
453	448	25:30/739	26:00/740	456
454	449	25:33/740	26:03/739	0
455	451	25:45/740	26:15/739	0
456	453	26:00/740	26:30/739	0

CARAGE DEADHEAD TIMES

FROM ANJOU CARAGE TO POINT 739: 15

FROM POINT 739 TO ANJOU CARAGE: 15

FROM ANJOU CARAGE TO POINT 740: 22

FROM POINT 740 TO ANJOU CARAGE: 20

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